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FRICTIONLESS CONTACT OF A RIGID CONICAL FRUSTUM INDENTING A TRANSVERSELY ISOTROPIC FUNCTIONALLY GRADED HALF SPACE

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ABSTRACT

This paper is concerned with the problem of frictionless contact between a rigid axially symmetric conical frustum and an elastic half space of transversely isotropic and functionally graded material. The frustum is under the action of axial load causing it to press the half space normally. The investigation focuses on determining the contact pressure, contact area and the load-contact area relationship. The solution of the mathematical model corresponding to this problem has been e effected through use of Hankel transforms of different order on field equations and reducing the system of equations to a Cauchy type singular integral equation. Applying standard numerical method the equations are solved and the results of the investigation are displayed graphically.

Key words and phrases: functionally graded material, transversely isotropic medium, Hankel transform, singular integral equation.

1. INTRODUCTION

After the initial investigation of Hertz in 1882 contact problem got its immense importance in the study of solid mechanics problems. One of the reasons behind is the fact that contact is the principal method of applying loads to a deformable body and the resulting stress concentration is often the most critical point in the body. With the application of load, contact area may or may not vary; accordingly contact problems have been classified as advancing (increase of contact area), receding (decrease of contact area) and stationary (contact area remaining the same). In the case of advancing or receding contact, determination of contact area is an additional task barring the determination of contact pressure distribution, which has always been of great interest to engineers. Another kind of classification of contact problems is also made based on the presence or absence of frictional force at the surface of contact. However, in our present discussion we shall consider frictionless contact problem ignoring any friction at the surface of contact between the bodies. There are quite a good number of contact problems investigated in literature in recent past where the frictional forces at the contact surface have been ignored. Among the several works done, we may mention a few: Shvets et al. [1], Paris et al.[2], Barik et al. [3, 4], Birinci and Erdol [5], Kit and Monastyrsky [6], Fabrikant [7] etc.

Earlier investigations of contact problems were mostly confined to isotropic and homogeneous bodies only. But over the last few decades various problems in solid mechanics are being studied considering the solid as functionally graded (isotropic or anisotropic). Functional gradation leads to non-homogeneity of the medium in respect to elastic properties. The elastic coefficients are no longer constants everywhere in the medium, rather they are either sectionally constants or are functions of position. The idea of non-homogeneity is not at all hypothetical, but more realistic. Elastic properties in soil may vary considerably with coordinates. The earth crust itself is non-homogeneous. Besides these, lots of structural materials are considered as functionally graded materials (FGMs) with distinct non-homogeneous character. For example, in graded composite materials, graded regions are treated as series of perfectly bonded composite layers, each layer being assigned slightly different elastic properties. Thus study of solid mechanics problems should not be restricted to isotropic homogeneous medium only. The study needs to be extended to the anisotropic and functionally graded materials also. Among the recent works on the contact problems mention may be made of the works of Barik et al. [8], Dag [9], Ke et al. [10], Wang et al. [11, 12] etc. The analytical studies involving receding contact in homogeneous and graded media can be found in the papers by Garrido et al. [13], Garrido and Lorenzana [14], Chaudhuri and Ray [15, 16], Comez et al. [17], El-borgi et al. [18] etc. Owing to their applications in a great variety of structural systems, such as foundations, pavements in roads and runways, automotive disk brake systems and many other technological applications, considerable progress has been made with the analysis of contact problems in the theory of elasticity.

The present investigation aims to and the elastostatic solution of an axially symmetric frictionless contact between a transversely isotropic functionally graded half space and a rigid axially symmetric conical frustum loaded by a concentrated force P along the axis. The graded half space is modeled as a non-homogeneous medium with exponential variations of the elastic coefficients in a direction perpendicular to the plane of isotropy. The study of this problem was motivated by the need for better understanding of a dynamic micro-contact printing process [19], where PDMS (polydimethylsiloxane) stamps were used to print on a substrate. Each of these stamps can be modeled as a conical frustum, while the substrate can be represented as a transversely isotropic elastic material. In our method of solution we have used integral transform technique to reduce the considered problem to a problem of Cauchy type singular integral equation, which has been solved numerically. Numerical computations have been done to assess the effects of graded parameters considered in the problem on various subjects of interest and the results have been shown graphically.

2. Formulation of the Problem

A rigid axisymmetric conical frustum is supposed to be lying on an elastic half space of transversely isotropic and functionally graded material and pressing the half space normally after being acted upon by an applied load P on it(Fig. 1). Let be the indentation depth of the punch and f(r) be the punch pro le represented by the equations

$$f(r) = 0, \qquad 0 \le r \le b$$
$$= (r-b)\cot\emptyset \qquad b \le r \le a \tag{2}$$

where $\emptyset$ is the semi vertical angle of the cone, b is the radius of the at end area of the punch and a is the radius of the contact area. We shall assume that the axis of symmetry of the frustum is parallel to the line of symmetry of the transversely isotropic material. As a reference frame we choose cylindrical coordinate system (r; ; z) to specify the position of a point in the half space. The origin is taken at the free surface of the half space with z-axis pointing into the medium and coinciding with the axis of symmetry of the frustum. Due to axisymmetric nature of the material as well as of applied load, the field variables are all independent of and the displacement vector (u, 0, w) will be functions of r and z only. The strain displacement relations (Lekhnitskii, [20]) are respectively given by

$$e_{rr} = \frac{\partial u}{\partial r}, \qquad e_{\theta\theta} = \frac{u}{r}, \qquad e_{zz} = \frac{\partial w}{\partial z}$$
$$e_{zz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right), \tag{3}$$

$$e_{\theta z} = e_{r\theta} = 0$$

$$\sigma_{rr} = C_{11}e_{rr} + C_{12}e_{\emptyset\emptyset} + C_{13}e_{zz}$$

$$\sigma_{\theta\theta} = C_{12}e_{rr} + C_{11}e_{\emptyset\emptyset} + C_{13}e_{zz}$$

$$\sigma_{zz} = C_{13}e_{rr} + C_{163}e_{\emptyset\emptyset} + C_{33}e_{zz} \qquad (4)$$

$$\sigma_{rz} = 2C_{44}e_{rz}, \quad \sigma_{\theta z} = 2C_{44}\, e_{\theta z}$$

$$\sigma_{r\theta} = (C_{11} - C_{12})e_{r\theta}$$

The functional grading of the material of the half space is assumed to be in the direction of z-axis such that the elastic coefficients C_{ij} vary exponentially according to the law

$$C_{ij}\,(z) = A_{ij}\exp(\alpha z)\,, \qquad 0 < z < \infty \qquad (5)$$

where A_{ij} are the anisotropic coefficients in the homogeneous medium and is the no homogeneity parameter controlling the variation of the anisotropic coefficients in the graded medium. It follows from equations (3) and (4) that $\sigma_{\theta z} = 0$, $\sigma_{r\text{-}\theta} = 0$, $\sigma_{ij} = \sigma_{ij}\,(r, z)$ for the other stress components. As a result, the equilibrium equation (in absent of body forces) for the present axisymmetric problem are given by

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{r}\sigma_{rz} = 0 \qquad (6)$$

Using relationships (3), (4) and (5) in the equilibrium equations (6) and considering the nature of the punch, the mathematical formulation of the present problem consists of

(i) **Equilibrium equations :**

$$\frac{\partial}{\partial r}\left[\frac{1}{r}(ru)\right] + (A_{13} + A_{44})\frac{\partial^2 w}{\partial r\,\partial z} + A_{44}\frac{\partial^2 u}{\partial z^2} + \alpha\, A_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) = 0, \qquad (7a)$$

$$A_{44}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + (A_{13} + A_{44})\frac{\partial}{\partial z}\left[\frac{1}{r}\frac{\partial}{\partial r}\,(ru)\right] + A_{33}\,\frac{\partial^2 w}{\partial z^2} + \alpha\left[A_{13}\frac{1}{r}\frac{\partial}{\partial r}(ru) + A_{33}\frac{\partial w}{\partial z}\right] = 0 \qquad (7b)$$

Boundary conditions:

$$\sigma_{zz}(r,0) = 0), \qquad r > a \qquad (8a)$$
$$\sigma_{rz}(r,0) = 0, \quad r \geq 0 \qquad (8b)$$
$$w(r,0) = \delta - f(r), \quad 0 \leq r < a \qquad (8c)$$

3. Method of solution:

Application of rest order Hankel transform to equation (7a) and zeroth order Hankel transform to (7b) over the variable r transforms the equations (7) to

$$A_{44}\frac{\partial^2 \bar{u}^1}{\partial z^2} - s^2\bar{u}^1 - (A_{13} + A_{44})s\frac{\partial \bar{w}^0}{\partial z} + \alpha A_{44}\left(\frac{\partial \bar{u}^1}{\partial z} - s\bar{w}^0\right) = 0 \qquad (9a)$$
$$A_{33}\frac{\partial^2 \bar{w}^0}{\partial z^2} - A_{44}s^2\bar{w}^0 + (A_{13} + A_{44})s\frac{\partial \bar{u}^1}{\partial z} + \alpha\left[A_{13}s\bar{u}^1 + A_{33}\frac{\partial \bar{w}^0}{\partial z}\right] = 0 \quad (9b)$$

where s is the transform parameter and superscript 1 is used to represent first order Hankel transform while superscript 0 for zeroth order Hankel transform. To solve the system of homogeneous equations (9) we take

$$\bar{u}^1 = A\,e^{-\lambda s z}$$
$$\bar{w}^0 = B\,e^{-\lambda s z} \qquad (10)$$

where A and B are constants and λ is a parameter. Substitution of (10) into (7) leads to the characteristic equation for the determination of the parameter λ

$$(s\lambda^2 A_{44} - \alpha\lambda A_{44} - s)(sA_{44} + \alpha\lambda A_{33} - s\lambda^2 A_{33})$$
$$-\{\lambda s(A_{13} + A_{44}) - \alpha A_{13}\}\{\lambda s(A_{13} + A_{44}) - \alpha A_{44}\} = 0 \qquad (11)$$

The equation (11) can be written as a quadratic equation in $(\lambda^2 s - \alpha\lambda)$ and obviously the nature of the roots depend upon A_{ij} and α .Corresponding to each root we obtain two values of λ . For the existence of solution $\bar{u}^1$ and $\bar{w}^0$ $at\ z \to \infty$, the roots λ_1 and λ_2 are taken with positive real parts. For the root λ_1 the constants in (10) are taken as A_1, B_1 and corresponding homogeneous equation of (9b) gives the following relation between A_1 and B_1

$$A_1 = G(\lambda_1)B_1 \qquad (12)$$

where $$G(\lambda_1) = \frac{s\lambda_1^2 A_{33} - sA_{44} - \alpha\lambda_1 A_{33}}{\lambda_1 s(A_{13} + A_{44}) - \alpha A_{13}} \qquad (13)$$

If all of the foregoing subscripts 1's are replaced by 2's then the resulting equation becomes a new relation between A_2 and B_2 for the second root λ_2. Thus the transformed displacement and stress components can be written in the following form

$$\bar{u}^1(s,z) = A_1\,e^{-\lambda_1 s z} + A_2\,e^{-\lambda_2 s z}$$
$$\bar{w}^0(s,z) = B_1\,e^{-\lambda_1 s z} + B_2\,e^{\lambda_2 s z} \qquad (14)$$

$$\bar{\sigma}^1_{rz}(s,z) = A_{44}\left(\frac{\partial \bar{u}^1}{\partial z} - s\bar{w}^0\right)$$
$$\bar{\sigma}^0_{zz}(s,z) = A_{13}s\bar{u}^1 + A_{33}\frac{\partial \bar{w}^0}{\partial z}. \qquad (15)$$

Taking Hankle Transform of order zero and one respectively for the boundary condition (8a), 8(b) and using equation (14) we have

$$A_{13}(A_1 + A_2) - (A_{33}(\lambda_1 B_1 + \lambda_2 B_2) = g(s) \qquad (16)$$

And

$$\lambda_1 A_1 + \lambda_2 A_2 + B_1 + B_2 = 0 \qquad (17)$$

where g(s) is Henkel Transform of unknown function defined as $g(r) = 0,\ r > a$.Utilization of (12) into equations (16),(17) yields

$$B_i = (-1)^i \ \frac{g(s)\{1+\lambda_{i+1}G(\lambda_{i+1})\})}{\Delta},\ i = 1{,}2 \qquad (18)$$

where

$$\Delta = \{G(\lambda_2) - G(\lambda_1)(\lambda_1\lambda_2 A_{33} + A_{13}) + A_{33}(\lambda_1 - \lambda_2) + G(\lambda_1)G(\lambda_2)A_{13}(\lambda_1 - \lambda_2)\}$$

Substitution of equation (18) and second equation of (14) into the Henkel Transformed equation (8c) leads to a Cauchy type singular integral equation

$$\int_0^a [\frac{1}{t-r} + L(r,t)]g(t)dt = M(r), 0 < r < a \qquad (19)$$

where $L(r,t) = \chi t L_0(r,t) + t L_1(r,t) - \frac{1}{t-r}$, (20)

$$L_0(r,t) = \int_0^\infty J_0(st)J_2(rs)ds = \frac{1}{r(r^2-t^2)}, 0 < t < r$$
$$=0, \quad 0 < r < t \qquad (21)$$

$$L_1(r,t) = \int_0^\infty [N(s) - \chi]J_0(st)J_2(rs)ds, \qquad (22)$$

$$N(s) = \frac{\{\lambda_1 G(\lambda_1) - \lambda_2 G(\lambda_2)\}}{\Delta} \qquad (23)$$

χ is the limiting value of N(s) as $s \to \infty$. The expressions of χ is given by

$$\chi = \frac{2\left(\sqrt{R_1} + \sqrt{R_5}\right)}{\sqrt{R_1 R_5}(A_{13} - A_{33})} \qquad (24)$$

where $R_1 = 4\left(R + \sqrt{Q_1}\right), R_5 = 4\left(R - \sqrt{Q_1}\right),\ R = \frac{A_{33} - A_{13}^2 - 2A_{13}A_{44}}{2A_{33}A_{44}}$,

$$Q_1 = R^2 - \frac{1}{A_{33}}.$$

Using the substitution $t = (1+u)\frac{a}{2}, r = (1+x)\frac{a}{2}$ and the notations $g^*(u) = g[(1+t)\frac{a}{2}]$ and $M^*(x) = M[(1+r)\frac{a}{2}]$.

The equation (19) is transformed into a singular integral equation

$$\int_{-1}^{1} g^*(u)\left[\frac{1}{u-x} + \frac{a}{2}L^*(u,x)\right]du = M^*(x) \qquad (25)$$

where $L^*(u,x) = L\left[(1+u)\frac{a}{2}, (1+x)\frac{a}{2}\right]$

where $$L^*(u,x) = L\left[(1+u)\frac{a}{2}, (1+x)\frac{a}{2}\right]$$

The integral equation (25) can be evaluated by using Gauss-Chebyshev method (21). Finally using (12), (18) into second equation of 15) and taking Hankel inversion we get the expression $\sigma_{zz}(r,0)$:

$$\sigma_{zz}(r,0) = \int_0^\infty Q\ (\lambda,s)\bar{g}^0(s)sJ_0(rs)ds \qquad (26)$$

Where

$$Q(\lambda,s) = \frac{(A_{13}G(\lambda_2) - A_{33}\lambda_{22})\{1+\lambda_1 G(\lambda_1)\} - (A_{13}G(\lambda_1) - A_{33}\lambda_1)\{1+\lambda_2 G(\lambda_{21})\}}{\Delta}$$

Now equilibrium condition demands

$$P + 2\pi \int_0^a [r\sigma_{zz}(r,0)\}]dr = 0 \qquad (27)$$

The equation (27) is the relationship between the applied load P and the radius of the contact area. As σ_{zz}(r; 0) is given by (26) is in the form of integral with no closed form expression, the relationship needs to be evaluated numerically.

6. Numerical results and discussions:

The present study is related to the study of axially symmetric frictionless contact between a transversely isotropic elastic functionally graded half-space and a rigid conical frustum. The main objective of the present discussion is to study the effects of graded parameters on the contact problem. The presence of the graded parameter in FGMs makes the governing differential equations more complex to get a complete analytical solution. Solution of the problem can be obtained using numerical methods. Following the standard numerical method normal stress and applied normal load have been computed numerically through equations (25)-(27) and are shown graphically. In the present discussion we have taken the values of elastic coefficients for magnesium (Baric et al. [22]) as

$$A^0_{11} = 5.97,\ A^0_{12} = 2.62,\ A^0_{13} = 2{:}17,\ A^0_{33} = 6.17,\ A^0_{44} = 1.64.$$

Figures 2-5 show the variation of normal stress distribution with radial distance r. Fig. 2 shows the effect of graded parameter on normal stress σ_{zz} (r, 0). It is observed that as decreases (which effectively means here that the rigidity of the material decreases), the normal stress also decreases for a particular value of r. This is expected from the physical point of view in the sense that material with lower rigidity can offer lower resistance to an applied load. Fig. 3 exhibits the influence of the semi vertical angle of the conical frustum on normal stress σ_{zz} (r,0). It is clear from Fig. 3 that normal stress σ_{zz} (r, 0) is increasing with the decreasing values of $\emptyset$, the semi vertical angle of the conical frustum. Effect of b, the radius of the flat ended indentor is shown in Fig. 4. As expected, the result shows that stress distribution increases with the increase of b. It is observed from Fig. 5 that normalized stress distribution is increased with the increased values of the indentation depth of the conical frustum. There is one point to note here that for all the above cases for any particular value of any one of the parameters of α, δ, $\emptyset$ and b, the values of normal stress σ_{zz} (r, 0) is increasing with the increasing values of r with maximum slope being near the origin.

Relation between applied normal load P and contact radius a are depicted in Figures 7-9. It is observed that the values of applied load P increases proportionally with contact radius a in all cases and effects of various parameters on P- a relation are quite clear in figures 6-9.

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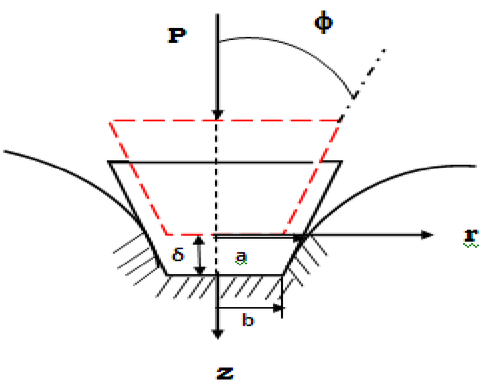

Fig. 1 Geometry of the problem

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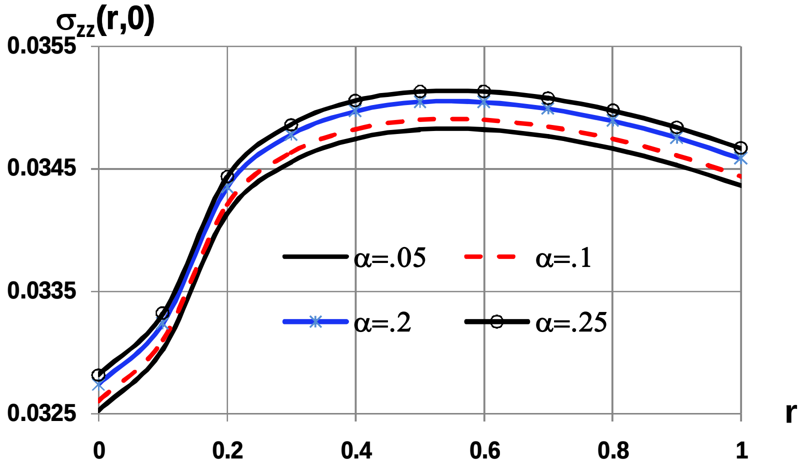

Fig . 2 Effect of graded parameter α of indentor on $\sigma_{zz}(r,0)$

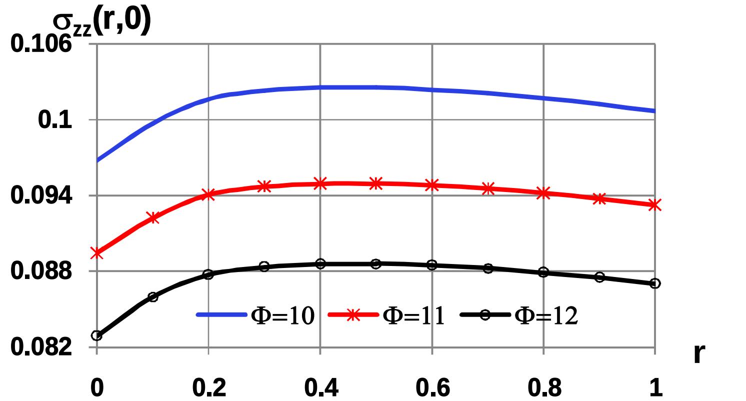

Fig 3 Effect of semi vertical angle of indentor on $\sigma_{zz}(r,0)$

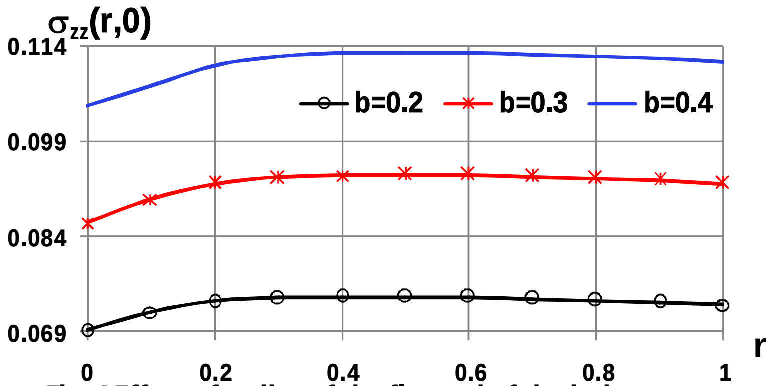

Fig. 4 Effect of radius of the flat end of the indentor on $\sigma_{zz}(r,0)$

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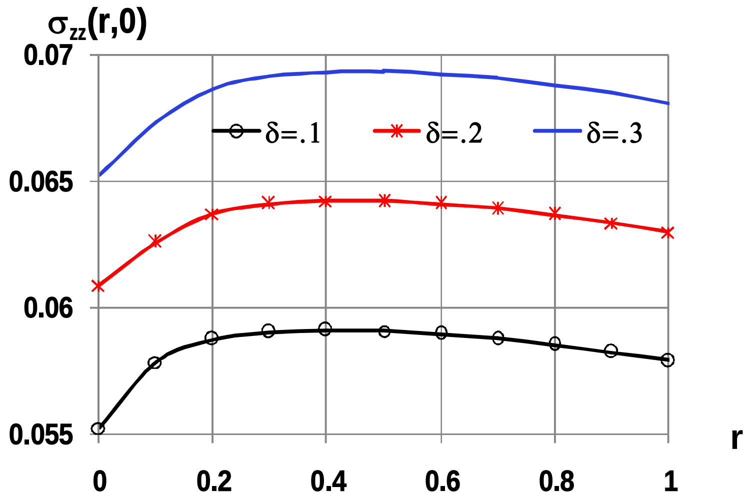

Fig. 5 Effect of indentation depth δ of indentor on $\sigma_{zz}(r,0)$

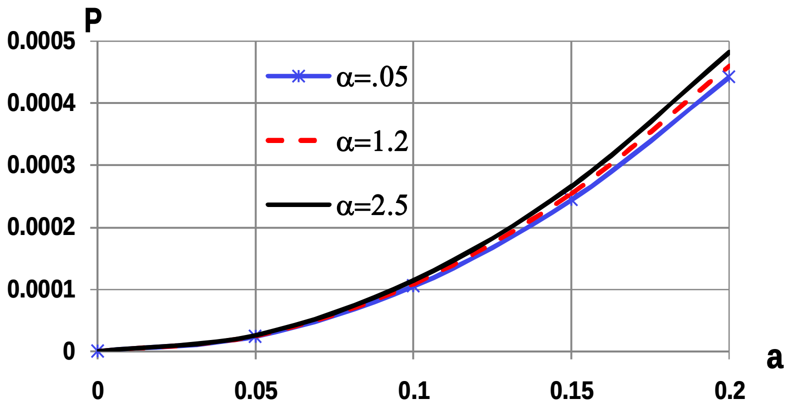

Fig. 6 Effect of grded parameter α on applied load P

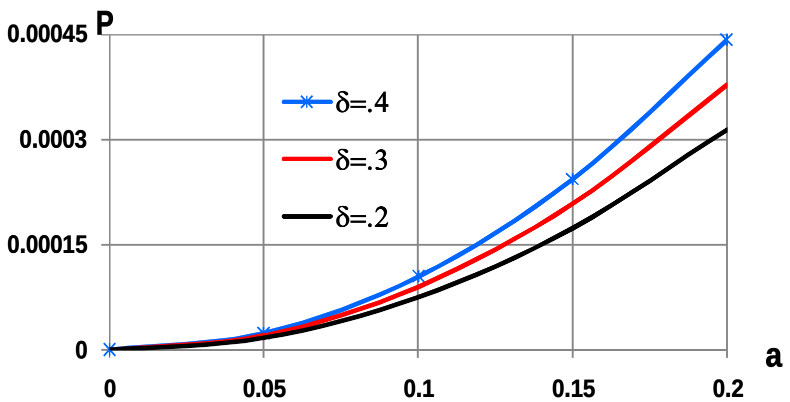

Fig. 7 Effect of indentation depth δ on applied load P

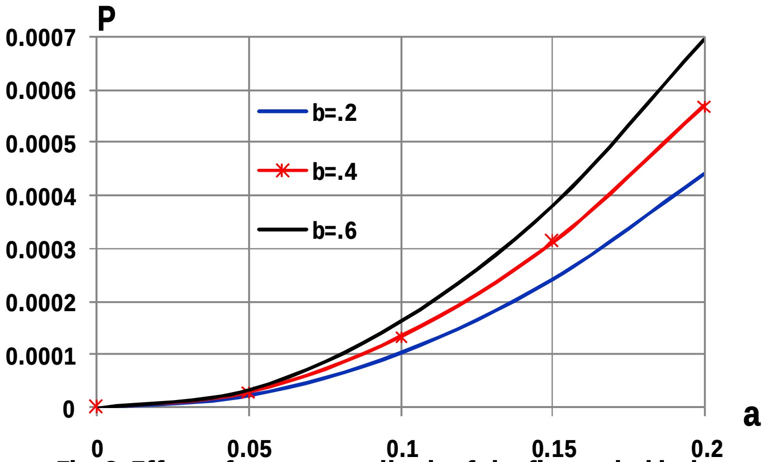

Fig. 8 Effect of contact radius b of the flat ended indentor on applied load P

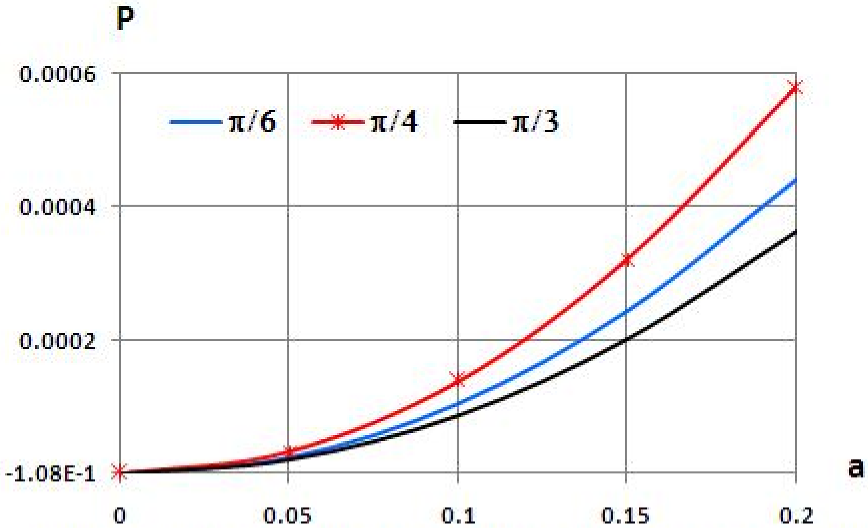

Fig 9 Effect of semi vertical angle Φ of indentor on applied load P