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Analysis on the Dynamic Effect of Shear Force on a Rectangular Block

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ABSTRACT

The Paper presents an analysis on the dynamic effect of shear force on a rectangular block free to move and thereafter the concept is incorporated on a cast iron block to study whether the theoretically calculated results corresponds to the experimental results. The paper also presents the conditions for toppling and failure when the shear force is applied on the extreme end of the rectangular block. Although the experiment has be carried out for a cast iron block, the main goal of this experiment is to analyze the effect of shear force on large structures, beams and truss members. With further optimization this process can also be utilized to calculate the young's modulus of a material.

Keywords:- Shear Force, Toppling, Fracture, Stress Intensity Factor

1. INTRODUCTION

The mathematical treatment on the equilibrium of cantilever beams or any other beams subjected to load does not involve a great difficulty [1-4]. Nevertheless, unless small deflections are considered, an analytical solution does not exist, since for large deflections a differential equation with a non-linear term must be solved. The equation is said to involve geometrical non-linearity [5, 6]. An excellent treatment of the problem of deflection of beam, built-it at one end and loaded at the other with a vertical concentrated force, can be found in "The Feynman Lectures on Physics" [2], as well as in other university textbooks on physics, mechanics and elementary strength of materials. The analysis of large deflections of cantilever beams of elastic material can be found in Landau's book on elasticity [5], and the solution in terms of elliptic integrals was obtained by Bisshopp and Ducker [7].

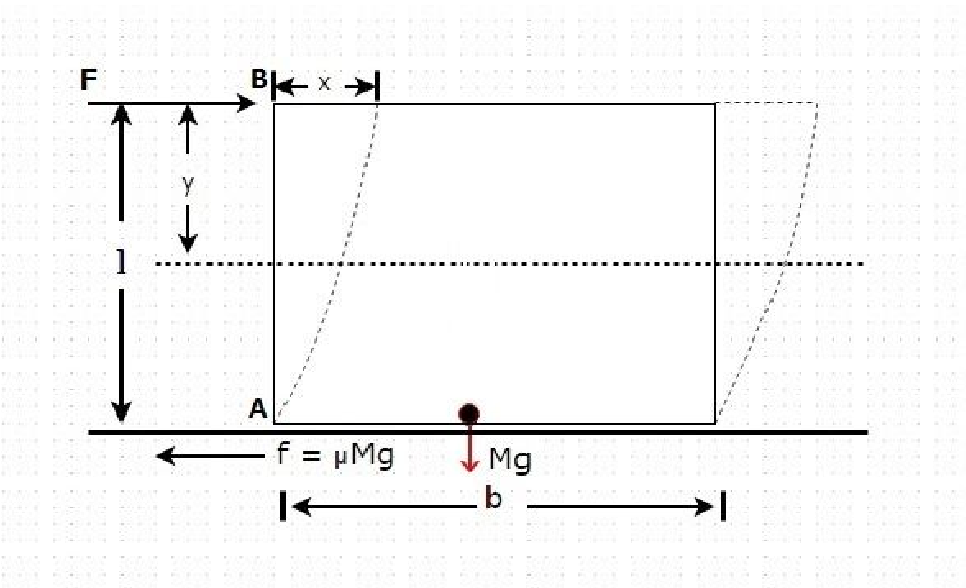

Figure 1: Effect of Shear Force on a Rectangular Block

Moreover, with some modifications on the mathematical treatment for bending of beams, we can derive the mathematical expression for bending of a body subjected only to friction. From our general concept of physics and elementary strength of materials, we know that when the body is subjected to a shear force above the centroidal axis, it first undergoes a deformation and then comes into motion as shown in Figure 1. However, for such deformation, the force must be a shear force acting above the centroidal axis, which lies at the base of the rectangular block i.e. at the interface of the two surfaces. In such case, the body can be considered as a cantilever beam before the limiting friction is reached and with the help of certain assumptions the mathematical modeling can be done and the additional force can be calculated.

2. METHODOLOGY

2.1. Mathematical modeling

Let us consider the following nomenclature for deriving the mathematical expression for the additional force.

E – Young's modulus of the material
I – Moment of inertia of the material
F – Shear force
F_f – Frictional force
F_t – Total force
M – Mass of the body
x – Linear deformation
y – Depth from the top of the block
l – Height of the block
b – Width of the block
μ - Coefficient of friction between the block and the surface
g – Acceleration due to gravity

Assuming the rectangular block to be homogeneous and isotropic and considering that the centre of gravity lies on the centroidal axis, we have

$$EI\,\frac{d^2x}{dy^2} = Fy - \mu Mgy$$

$$=> EI\,\frac{dx}{dy} = \frac{Fy^2}{2} - \frac{\mu Mgy^2}{2} + C_1$$

$$At\ A,\quad \frac{dx}{dy} = 0\ and\ putting\ y = l$$

$$=> C_1 = \frac{\mu Mgl^2 - Fl^2}{2}$$

$$=> EI\,\frac{dx}{dy} = \frac{Fy^2}{2} - \frac{\mu Mgy^2}{2} + \frac{\mu Mgl^2 - Fl^2}{2}$$

$$=> EIx = \frac{Fy^3}{6} - \frac{\mu Mgy^3}{6} + \frac{\mu Mgl^2y - Fl^2y}{2} + C_2$$

$$At\ A,\quad x = 0\ and\ putting\ y = l$$

$$=> C_2 = \frac{\mu Mgl^3 - Fl^3}{6} - \frac{\mu Mgl^3 - Fl^3}{2} = \frac{Fl^3 - \mu Mgl^3}{3}$$

$$Maximum\ value\ of\ x\ will\ occur\ at\ point\ B\ due\ to\ maximum\ deformation\ due\ to\ bending.$$

$$\therefore At\ B, x = x_{max}\ and\ y = 0$$

$$=> EIx_{max} = \frac{Fy^3}{6} - \frac{\mu Mgy^3}{6} + \frac{\mu Mgl^2y - Fl^2y}{2} + \frac{Fl^3 - \mu Mgl^3}{3} = \frac{Fl^3 - \mu Mgl^3}{3}$$

$$=> x_{max} = \frac{Fl^3 - \mu Mgl^3}{3EI}$$

$$Again, Fl^3 = \mu Mgl^3 + 3EIx_{max}$$

$$=> F = \mu Mg + \frac{3EIx_{max}}{l^3}$$

$$For\ a\ rectangular\ block, we\ have\ I = \frac{1}{12}\,bl^3$$

$$\therefore F = \mu Mg + \frac{Ebx_{max}}{4}$$

$$\therefore Additional\ force\ to\ move\ the\ body, F_{ad} = F - \mu Mg = \frac{Ebx_{max}}{4}$$

From the above derived equation, we can infer that the rectangular block will topple when $x_{max} > b$ i.e.

$$F > \frac{Eb^2}{4} + \mu Mg$$

Shear stress developed in the block is

$$\sigma_s = E\,\epsilon^p = \frac{Ex^p}{b^p}\text{, where } p \text{ is the stran index that depends upon the plasticity of the body.}$$

Fatigue toughness is an indication of the amount of stress required to propagate a pre-existing flow. It is a very important material property since the occurrence of flow is not completely available in the processing fabrication, or service of a material. A parameter called stress-intensity factor (k) is used to determine the fracture toughness of most materials. It is used in fracture mechanics to predict the stress state near the tip of a crack caused by a remote load or residual stresses [8]. The stress intensity function is a function of loading crack size and structural geometry.
The stress intensity factor may be represented by the following equation:

$$k = \sigma_s\sqrt{\pi c}\text{, where } c \text{ is the crack length [9]}$$

Hence, the material will just fail when

$$k = \lim_{c\to 0}\sigma_s\sqrt{\pi c}$$

$$=> \sigma_s = \lim_{c\to 0}\frac{k}{\sqrt{\pi c}}$$

$$=> \frac{Ex^p}{b^p} = \lim_{c\to 0}\frac{k}{\sqrt{\pi c}}$$

2.2Experimental Details

Although this research finds its application in the analysis of large structures, a small cast iron block is chosen as a specimen to investigate the validity of the mathematical modeling. An experimental setup was made with the combination of a dial indicator and a dynamometer. The tool attached to the dynamometer is slowly fed into the metal block and the dial indicator measures the deflection on the cast iron block until the limiting friction is reached and the body just tends to slip on the surface. A proximity sensor is used to detect the slightest movement of the block on the surface. The net equivalent force is simultaneously measured and displayed on the digital screen of the dynamometer. The frictional force between the specimen and the surface is subsequently subtracted from the total force measured by the dynamometer to obtain the value of the additional force. The experimental and theoretical values of the additional forces are tallied to investigate the validity of the mathematical model.

3.RESULTS AND DISCUSSION

In this investigation, a cast iron specimen is selected as the work piece. The specification of the test specimen is represented in Table 1 whereas Table 2 shows the results obtained from the experiment conducted on the basis of the aforementioned methodology.

Table 1: Specification of the Cast Iron Block

Mass (M)	Young's Modulus (E)	Height (l)	Width (b)
768.4 gm	110 GPa	48.20 mm	47.16 mm

Table 2: Experimental Result on a Cast Iron Block

SL No.	y (mm)	x (µm)	F_f (N)	F_t (N)	Additional Force, F_{ad} (N)	
					Experimental $F_{ad} = F_t - F_f$	Theoretical $F_{ad} = \frac{Ebx}{4}$
1	20	0.010	3.78	15.76	11.98	12.97
2	15	0.012	3.78	19.86	16.08	15.56
3	10	0.015	3.78	23.00	19.22	19.45
4	5	0.018	3.78	25.56	21.78	23.34
5	1	0.020	3.78	31.26	27.48	25.94

It can be clearly noticed from the result that the deflection is in the micro order and the entire experiment has to be carried out with highly sensitive mechanical instruments. However, the minor difference in the results of the experimentally obtained additional force and the theoretical obtained additional force can be explained in a number of ways. The instruments required for measuring the force and the deformation have to be very sensitive since the deformations are obtained in the micro order and the additional force is also quite small. A little error in the reading will lead the results to vary to a large extent. Moreover, the specimen is not absolutely homogenous and isotropic mainly due to casting defects and impurities. However, both the result obtained closely corresponds to one another and hence the mathematical model is valid.

4.CONCLUSION

From the above investigation, we can conclude that the additional force F_{ad} depends on the elasticity of the work piece material and the width of the block. The deformation on the block will however depend on the elasticity of the material. For a perfectly rigid body, x_{max} is zero. Hence the same amount of force, when applied at any point, will bring the body into motion. The graph displaying the variation of the additional force with depth y has been shown in Figure 2. It is quite evident from the plot that the additional force required to displace the block is maximum at the top of the block and it gradually decreases as the distance of the point of force increases from the top of the rectangular block.

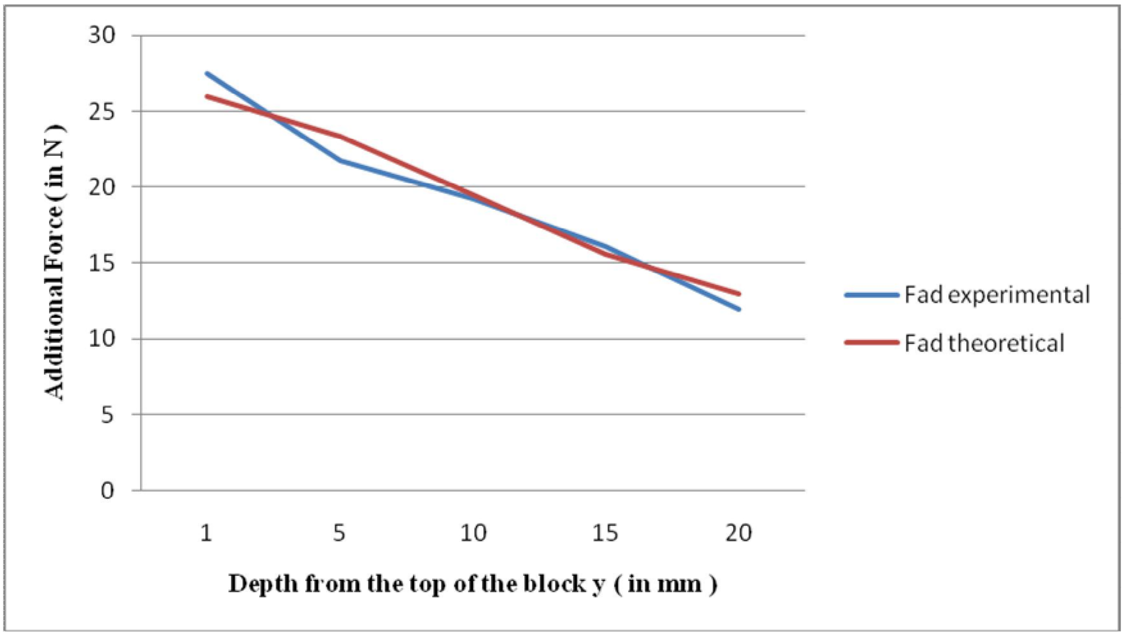

Figure 2: Variation of F_{ad} with vertical depth y

With a little modification for different geometry and shape of the material, this research can find its application in the analysis of large structures, truss members, bridges and other movable beams. However the same experimental setup can be used to determine the elasticity of a material.

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