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Fixed-Time Consensus Tracking of Second-Order Multi-Agent Systems with Bounded Disturbances

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ABSTRACT

This paper investigates the fixed-time consensus tracking problem of second-order multi-agent systems with bounded disturbances. Suppose that the communication topology is described by a weighted undirected graph. A novel nonlinear control protocol is proposed to solve the fixed-time consensus tracking problem of second-order multi-agent systems with the help of sliding mode technique and the sufficient conditions are obtained by theoretical analysis. Furthermore, the designed protocol can be robust against bounded disturbances influencing the agents and the upper bound of settling time can be estimated in advance without relying on initial conditions. Finally, some numerical examples are provided to verify the performance and effectiveness of the designed protocol.

Keywords: bounded disturbances, fixed-time consensus tracking, second-order multi-agent systems, sliding mode technique

1. INTRODUCTION

In recent years, distributed coordinated control of multi-agent systems has received considerable attention due to its wide application, such as formation control of multiple aircrafts, multi-robot aggregation, satellite attitude coordination and so on. Consensus is a key problem in distributed coordinated control of multi-agent systems. The so-called consensus is to design control protocols for each agent based on only local information from its neighbors such that the states of all agents converge to a common value. Convergence rate is an important performance index for researching the effectiveness of the proposed consensus protocol. Most of the previous literatures were about asymptotic convergence ([1-3]), that is, when settling time goes to infinity, consensus is achieved. In practical applications, control accuracy is very important, which may need to solve the consensus problem in a finite time. Compared with the asymptotic consensus, the finite-time consensus ([4-5]) can not only improve the convergence rate, but also has stronger immunity and noise immunity. Wang et al. discussed the finite-time consensus problem of multi-agent systems and investigated both the directed interaction case and the undirected interaction case ([5]) by using the finite-time stability theory ([4]). However, the settling time function of finite-time consensus is related to the initial conditions of the agents. If the initial conditions are very large, the settling time tends to be infinite, and if the initial conditions are not known, the settling time cannot be estimated, which limits the practical application.

In order to solve this problem, the fixed-time stability theory ([6]) is proposed. The upper bound of the settling time can be estimated in advance, which is independent of the initial conditions. Leaderless fixed-time consensus protocols were designed for multi-agent systems with integrator-type dynamics in [7-14]. Hong et al. investigated the robust fixed-time consensus problem for multi-agent systems with nonlinear dynamics and uncertain disturbances, and designed three different protocols in [13], which can arrive at a conclusion that the simpler the design of the control protocol is, the more conservative the parameter design is. In [15-18], leader-follower fixed-time consensus problems were addressed for first-order multi-agent systems. Ning et al. solved more complex multi-leader situations by containment control and achieved the fixed-time containment control for single-integrator agents for the first time in [16]. Shang et al. considered the fixed-time group consensus problem and proposed the protocols in [17-18], which can enable multiple agents to achieve different consistent states finally. It is noted that the above results are devoted to the consensus control of first-order multi-agent systems. In reality, many system models consist of second-order dynamics, such as mobile robots and mechanical systems. Therefore, it is meaningful to research the second-order multi-agent systems. However, it is difficult to generalize the existing results to second-order multi-agent systems due to the nonlinear nature of fixed-time convergent controllers for first-order multi-agent systems. Therefore, there are few works to consider fixed-time consensus tracking problems for second-order multi-agent systems. In [19], a nonlinear control protocol was proposed to achieve the fixed-time consensus tracking for second-order multi-agent systems based on sliding mode control. However, the control input of each follower directly depends on the neighbors' inputs, and there may be a loop problem when there are cycles in the communication graph. Moreover, uncertain disturbances are not considered in the system. In [20], the fixed-time consensus tracking problem for second-order multi-agent systems with bounded disturbances was solved with the aid of sliding mode technique and Lyapunov theory. However, the proposed control protocol enables tracking errors to converge to zero in a finite time along the sliding surface rather than in a fixed time.

Inspired by the above literature, in the paper we consider the fixed-time consensus tracking problem for second-order multi-agent systems with bounded disturbances under fixed topology. The main contributions of this paper are as follows. Firstly, the upper bound of settling time can be estimated in advance without relying on initial conditions. Secondly, compared to the control protocols for first-order multi-agent systems in [7-18], the protocol designed in this paper solves the consensus tracking problem for second-order multi-agent systems with the aid of sliding mode technique and fixed-time stability theory. Thirdly, compared with the control protocol in [19], the protocol designed in this paper can eliminate exogenous disturbances. Finally, compared to the control protocol in [20], the protocol designed in this paper can guarantee that the tracking errors can slide along the surface to reach the origin in a fixed time.

The rest of the paper is arranged as follows. In Section 2, some preliminaries about algebraic graph theory and fixed-time convergence are provided, and the problem statement are presented. In Section 3, the main theoretical results are given. Numerical simulations are provided in Section 4. Finally, conclusions are drawn in Section 5.

2. PRELIMINARIES AND PROBLEM STATEMENT

a. Some notations

R represents the set of real numbers. R_+ represents the set of positive real numbers. R^N represents the N-dimensional Euclidean vector space. Define the upper right-hand Dini derivative of the function $\varphi : R \to R$ at the point $t \in R$ by $D^*\varphi(t)$, i.e., $D^*\varphi(t) := \limsup_{h\to 0^+} \left(\varphi(t+h) - \varphi(t)\right)/h$. P^T represents the transpose of a matrix P. $\|\cdot\|_1$ and $\|\cdot\|_2$ refer to the 1-norm and 2-norm of a vector, respectively. $\|z\|_p := \left(\sum_{i=1}^{N} |z_i|^p\right)^{(1/p)}$, $z \in R^N$, $p \in R_+$ and $1_N = [1,1,\ldots 1]^T$. We define $sig(x)^k = sgn(x)|x|^k$, where $x \in R$ and $sgn(x)$ is the sign function.

b. Algebraic graph theory

Consider multi-agent systems with one leader and N followers. We describe the communication relation among N followers by a weighted undirected graph $G = (V, E, A)$. $G = (V, E, A)$ is made up of three members, i.e., the set of nodes $V = \{v_1, v_2, \ldots v_N\}$, the set of edges $E \subseteq V \times V$, and the weighted adjacency matrix $A = [a_{ij}]$, where a_{ij} is nonnegative adjacency elements. An edge of G is expressed by $e_{ij} = (v_i, v_j)$. When $e_{ij} \in E$, the node v_i is considered to be a neighbor of v_j. The adjacency matrix A is defined such that $a_{ij} = 1$ for $e_{ij} \in E$ and $a_{ij} = 0$ otherwise. For an undirected graph, the nodes v_i and v_j can exchange messages with each other, i.e., $e_{ij} \in E \Leftrightarrow e_{ji} \in E$ and we have $a_{ij} = a_{ji}$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined by $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$, $l_{ij} = -a_{ij}, i \neq j$, which satisfies the fact that $\sum_{j=1}^{N} l_{ij} = 0$. Note that L of an undirected graph is symmetric and semipositive definite. A path between two distinct nodes v_i and v_j is a sequence of distinct edges $(v_i, v_{k1}),(v_{k1}, v_{k2}),\ldots(v_{kl}, v_j)$ in the graph. A graph is called to be connected if there is a path between any two distinct nodes in G. When the undirected graph is connected, then L will have a simple zero eigenvalue with the associated eigenvector 1_N, and all the other eigenvalues are positive real numbers. As is known that the second smallest eigenvalue of the Laplacian matrix L is called the algebraic connectivity. Next, we describe the communication relationship between one leader and N followers by another graph $\bar{G}$. Define a diagonal matrix $B = diag(b_1, b_2, \ldots, b_N)$, where $b_i = 1$ if the leader's information is available to the follower i, and $b_i = 0$ otherwise. Also, $H = L + B$ is defined associated with the graph $\bar{G}$. In the following, $\lambda_{\min}$ and $\lambda_{\max}$ are expressed as the minimum and maximum eigenvalues of H, respectively. Obviously, for a connected undirected graph, the eigenvalues of matrix H, which is positive-definite, are positive real numbers.

c. Fixed-time convergence

Consider the following dynamic system

$$\dot{x}(t) = f(t, x(t)), x(0) = x_0, t \in R_+ \tag{1}$$

where $x \in R^N$ and $f : R_+ \times R^N \to R^N$ is a nonlinear function. If f is discontinuous with respect to the state variable x, the solutions of (1) are defined in the sense of Filippov [21]. Assume that the origin is an equilibrium point of (1).

Definition 1 ([4]): The origin of system (1) is said to be a globally finite-time stable equilibrium point if it is globally asymptotically stable and any solution $x(t, x_0)$ of (1) can satisfy $x(t, x_0) = 0, \forall t \ge T(x_0)$, where $T(x_0): R^N \to R_+ \cup \{0\}$ is called the settling-time function.

Definition 2 ([6]): The origin of system (1) is said to be a globally fixed-time stable equilibrium point if it is globally finite-time stable and the settling-time function $T(x_0)$ is bounded, i.e., $\exists T_{\max} > 0 : T(x_0) \le T_{\max}, \forall x_0 \in R^N$.

Lemma 1 ([6]): Assume that there is a continuous radially unbounded function $V : R^N \to R_+ \cup \{0\}$ such that

(1) $V(x(t)) = 0$ if and only if $x(t) = 0$;

(2) Any solution $x(t)$ of system (1) satisfies the following inequality $D^* V(x(t)) \le -\left(\alpha V^p(x(t)) + \beta V^q(x(t))\right)^k$

for $\alpha, \beta, p, q, k > 0 : pk < 1, qk > 1$.

Then, the origin of system (1) is globally fixed-time stable, and the following estimate holds:

$$T(x_0) \le T_{\max} = \frac{1}{\alpha^k(1 - pk)} + \frac{1}{\beta^k(qk - 1)}, \forall x_0 \in R^N$$

Lemma 2 ([8]): Assume that there exists a continuous radially unbounded function $V : R^N \to R_+ \cup \{0\}$ such that

(1) $V(x(t)) = 0$ if and only if $x(t) = 0$;

(2)Any solution $x(t)$ of system (1) satisfies the following inequality $D^* V(x(t)) \le -\alpha V^p(x(t)) - \beta V^q(x(t))$ for $\alpha, \beta > 0, p = 1 - (1/\mu), q = 1 + (1/\mu), \mu \ge 1$.

Then, the origin of system (1) is globally fixed-time stable, and the following estimate holds:

$$T(x_0) \le T_{\max} = \frac{\pi\mu}{2\sqrt{\alpha\beta}}, \forall x_0 \in R^N$$

d. Problem statement

Consider multi-agent systems with N followers labelled as 1 to N and one leader. The dynamics of the i-th follower can be given as

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) + d_i(t) \end{cases}, i = 1, 2, \ldots N \tag{2}$$

where $x_i(t) \in R$ and $v_i(t) \in R$ denote the position and velocity of agent i, respectively. $u_i(t) \in R$ is the associated control input of agent i to be designed. Moreover, $d_i(t) \in R$ is the uncertain disturbance.

The dynamics of the leader can be given by

$$\begin{cases} \dot{x}_0(t) = v_0(t) \\ \dot{v}_0(t) = u_0(t) \end{cases} \tag{3}$$

where $x_0(t), v_0(t), u_0(t) \in R$ denote the position, velocity and input of the leader, respectively. To facilitate the following theoretical analysis, some assumptions are made as follows.

Assumption 1: The communication graph without leader is undirected and connected, and at least one follower is connected to the leader, that is, B is non-zero matrix.

Assumption 2: The uncertain disturbance $d_i(t)$ is uniformly bounded by $d_{\max}$, i.e., $|d_i(t)| \le d_{\max}, i = 1, 2, \ldots N$.

Assumption 3: The control input u_0 of the leader is bounded, that is, there exists a known positive constant l such that $|u_0| \le l$.

Remark 1: Note that the upper bound of the uncertain disturbance $d_i(t)$ and the control input u_0 of the leader can be obtained by a prior knowledge of the physical system.

With a given protocol $u_i(t)$, second-order multi-agent systems with bounded disturbances are said to achieve the fixed-time consensus tracking for any initial states, if the positions and velocities of followers can track the leader's position and velocity in a fixed time, i.e., $\begin{cases} \lim_{t \to T} |x_i(t) - x_0(t)| = 0 \\ \lim_{t \to T} |v_i(t) - v_0(t)| = 0 \end{cases}, i = 1, 2, \ldots N$ and $x_i(t) = x_0(t)$, $v_i(t) = v_0(t)$ when $t \ge T$ for all the followers with disturbances. Furthermore, the upper bound of settling time can be estimated in advance by a positive constant $T_{\max}$, i.e., $T < T_{\max}$.

In order to facilitate the proof of the main results, we need to provide the following lemma.

Lemma 3 ([16]): For any vector $x \in R^N$, if $p > r > 0$, where p and r are scalar constants, then the inequality $\|x\|_p \le \|x\|_r \le N^{\frac{1}{r} - \frac{1}{p}} \|x\|_p$ holds.

3. MAIN RESULTS

This section introduces a distributed fixed-time control protocol to address the fixed-time consensus tracking problem of second-order multi-agent systems (2) with a leader (3).

Consider the following control protocol motivated by the work in [20]:

$$\begin{aligned} u_i = & -\alpha sig\left\{(v_i - v_0) + k_1 sig\left[\sum_{j=0}^{N} a_{ij}(x_i - x_j)\right]^{\phi} + k_2 sig\left[\sum_{j=0}^{N} a_{ij}(x_i - x_j)\right]^{\varphi}\right\}^2 \\ & -\beta \operatorname{sgn}\left\{(v_i - v_0) + k_1 sig\left[\sum_{j=0}^{N} a_{ij}(x_i - x_j)\right]^{\phi} + k_2 sig\left[\sum_{j=0}^{N} a_{ij}(x_i - x_j)\right]^{\varphi}\right\} \\ & -k_1 \phi \left|\sum_{j=0}^{N} a_{ij}(x_i - x_j)\right|^{\phi - 1}\left[\sum_{j=0}^{N} a_{ij}(v_i - v_j)\right] - k_2 \varphi \left|\sum_{j=0}^{N} a_{ij}(x_i - x_j)\right|^{\varphi - 1}\left[\sum_{j=0}^{N} a_{ij}(v_i - v_j)\right] \end{aligned} \tag{4}$$

where $0<\phi<1$, $\varphi>1$, $k_1>0$, $k_2>0$, control gains α, β are positive constants and will be derived later.

The tracking errors are defined as $e_{xi}=x_i-x_0$, $e_{vi}=v_i-v_0$, and differentiate them as follows:

$$\begin{cases}\dot{e}_{xi}=\dot{x}_i-\dot{x}_0=v_i-v_0=e_{vi}\\ \dot{e}_{vi}=\dot{v}_i-\dot{v}_0=u_i+d_i-u_0\end{cases}, i=1,2,\dots N$$

According to the consensus tracking protocol (4) and the definition of matrix H, the tracking error system can be written as follows:

$$\begin{aligned}\dot{e}_v=&-\alpha sig\left\{e_v+k_1 sig\left(He_x\right)^{\phi}+k_2 sig\left(He_x\right)^{\varphi}\right\}^2-\beta\operatorname{sgn}\left\{e_v+k_1 sig\left(He_x\right)^{\phi}+k_2 sig\left(He_x\right)^{\varphi}\right\}\\&-k_1\phi\left|He_x\right|^{\phi-1}\left(He_v\right)-k_2\varphi\left|He_x\right|^{\varphi-1}\left(He_v\right)+d-u_0 1_N\end{aligned} \tag{5}$$

where $e_x=\left(e_{x1},e_{x2},\dots,e_{xN}\right)^T$, $e_v=\left(e_{v1},e_{v2},\dots,e_{vN}\right)^T$ and $d=\left(d_1,d_2,\dots,d_N\right)^T$.

Next, in order to analyze the stability of the closed-loop error system (5), motivated by the work in [20], a novel sliding mode manifold is developed as

$$S=e_v+k_1 sig\left(He_x\right)^{\phi}+k_2 sig\left(He_x\right)^{\varphi} \tag{6}$$

where $0<\phi<1$, $\varphi>1$, $k_1>0$, $k_2>0$.

Theorem 1: Consider second-order multi-agent systems (2) with a leader (3) under fixed undirected topology. Assume that Assumptions 1, 2 and 3 hold, and $\beta-d_{\max}-l>0$, $\alpha>0$. For any initial states, the distributed protocol (4) can guarantee the fixed-time consensus tracking with the settling time bounded by $T<T_{\max}=T_1+T_2$, where

$$T_1=\frac{\pi}{2\sqrt{\alpha N^{-0.5}\left(\beta-d_{\max}-l\right)}},\ T_2=\frac{2}{\bar{\alpha}\left(1-\phi\right)}+\frac{2}{\bar{\beta}\left(\varphi-1\right)},\bar{\alpha}=k_1\left(2\lambda_{\min}\right)^{\frac{\phi+1}{2}},\bar{\beta}=k_2 N^{\frac{1-\varphi}{2}}\left(2\lambda_{\min}\right)^{\frac{\varphi+1}{2}}$$

Proof: Substituting (6) into (5) obtains

$$\dot{e}_v=-\alpha sig\left(S\right)^2-\beta\operatorname{sgn}\left(S\right)-k_1\phi\left|He_x\right|^{\phi-1}\left(He_v\right)-k_2\varphi\left|He_x\right|^{\varphi-1}\left(He_v\right)+d-u_0 1_N \tag{7}$$

Differentiating (6) with respect to the velocity error system (7), one gets

$$\dot{S}=-\alpha sig\left(S\right)^2-\beta\operatorname{sgn}\left(S\right)+d-u_0 1_N \tag{8}$$

Based on the above analysis, it can be proved that the position error e_x and the velocity error e_v can arrive at the sliding mode surface S in a fixed time T_1.

Consider the Lyapunov function $V_1=\frac{1}{2}S^T S$ and taking the time derivative of V_1 can yield

$$\dot{V}_1=S^T\dot{S}=-\alpha S^T sig\left(S\right)^2-\beta S^T\operatorname{sgn}\left(S\right)+S^T d-S^T u_0 1_N$$

Under the conditions that $\left|d_i\left(t\right)\right|\le d_{\max}, i=1,2,\dots N$ and $\left|u_0\right|\le l$, we choose control gain β such that $\beta-d_{\max}-l>0$ and control gain $\alpha>0$. Based on Lemma 3, the following inequality can be given by

$$\dot{V}_1\le-\alpha N^{-(1/2)}\left\|S\right\|_2^3-\left(\beta-d_{\max}-l\right)\left\|S\right\|_1\le-\alpha N^{-(1/2)}\left\|S\right\|_2^3-\left(\beta-d_{\max}-l\right)\left\|S\right\|_2$$

In addition, based on the fact $\left\|S\right\|_2=\sqrt{2V_1}$, the following inequality holds:

$$\dot{V}_1\le-\alpha N^{-(1/2)}\left(\left\|S\right\|_2^2\right)^{3/2}-\left(\beta-d_{\max}-l\right)\sqrt{2V_1}=-\alpha N^{-(1/2)}\sqrt{8}V_1^{3/2}-\left(\beta-d_{\max}-l\right)\sqrt{2}V_1^{1/2}$$

According to Lemma 2, the upper bound of convergence time for system (8) can be estimated by the following formula

$$T_1=\frac{\pi}{2\sqrt{\alpha N^{-0.5}\left(\beta-d_{\max}-l\right)}} \tag{9}$$

Thus, the position error e_x and the velocity error e_v can arrive at the sliding mode surface S in a fixed time T_1. After that, $S=0$ can be always held. Next, we will prove that the tracking errors e_x and e_v can reach the origin in a fixed time T_2.

Note that on the sliding surface $S=0$, (6) becomes

$$e_v = -k_1 sig\left(He_x\right)^{\phi} - k_2 sig\left(He_x\right)^{\varphi} \tag{10}$$

Consider the Lyapunov function $V_2 = \frac{1}{2} e_x{}^T He_x$ and taking the time derivative of V_2 can lead to

$$\dot{V}_2 = e_x{}^T He_v = -k_1 \left(He_x\right)^T sig\left(He_x\right)^{\phi} - k_2 \left(He_x\right)^T sig\left(He_x\right)^{\varphi}$$

Based on Lemma 3, the following inequality can be obtained:

$$\dot{V}_2 \le -k_1 \left\|He_x\right\|_2^{\phi+1} - k_2 N^{\frac{1-\varphi}{2}} \left\|He_x\right\|_2^{\varphi+1} = -k_1 \left(\left\|He_x\right\|_2^2\right)^{\frac{\phi+1}{2}} - k_2 N^{\frac{1-\varphi}{2}} \left(\left\|He_x\right\|_2^2\right)^{\frac{\varphi+1}{2}} \tag{11}$$

In the sequel, it follows from the Courant-Fischer theorem ([22]) that $\left\|He_x\right\|_2^2 \ge \lambda_{\min}\left(H\right) e_x{}^T He_x = 2\lambda_{\min}\left(H\right) V_2$. Therefore, we have

$$\dot{V}_2 \le -k_1 \left(2\lambda_{\min}\right)^{\frac{\phi+1}{2}} V_2^{\frac{\phi+1}{2}} - k_2 N^{\frac{1-\varphi}{2}} \left(2\lambda_{\min}\right)^{\frac{\varphi+1}{2}} V_2^{\frac{\varphi+1}{2}}$$

Introduce the following notations:

$$p = \frac{\phi+1}{2},\ q = \frac{\varphi+1}{2},\ \bar{\alpha} = k_1 \left(2\lambda_{\min}\right)^{\frac{\phi+1}{2}},\ \bar{\beta} = k_2 N^{\frac{1-\varphi}{2}} \left(2\lambda_{\min}\right)^{\frac{\varphi+1}{2}}$$

Then the total derivative of the Lyapunov function satisfies the following inequality:

$$\dot{V}_2 \le -\bar{\alpha} V_2{}^p - \bar{\beta} V_2{}^q,\ \bar{\alpha}, \bar{\beta} > 0, 0 < p < 1, q > 1$$

Based on Lemma 1, the tracking errors e_x and e_v will converge to zero in a fixed time T_2 , which can be estimated by the following formula:

$$T_2 = \frac{2}{\bar{\alpha}\left(1-\phi\right)} + \frac{2}{\bar{\beta}\left(\varphi-1\right)} \tag{12}$$

The proof is completed.

Remark 2: It is proved that the trajectories of tracking errors e_x and e_v can reach the sliding mode surface S after $t \ge T_1$. Then, the tracking errors can slide along the surface to reach the origin after $t \ge T_1 + T_2$. Note that both T_1 and T_2 are independent of the initial conditions, which can be designed in advance by properly choosing the control parameters.

4. NUMERICAL SIMULATIONS

In this section, a numerical example is provided to show the effectiveness of our theoretical results. Consider the second-order multi-agent systems consisting of one leader and three followers and the communication topology is shown in Figure 1.

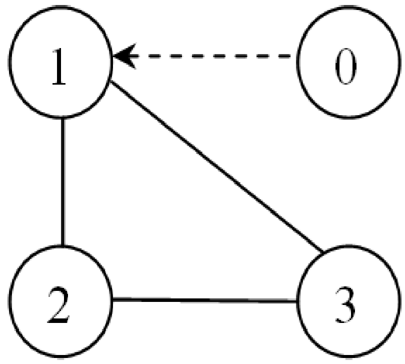

Figure 1 Communication topology

The state of leader is considered as $x_0 = \sin(0.5t)$, then $v_0 = 0.5\cos(0.5t), u_0 = -0.25\sin(0.5t)$. Hence, let $l = 0.25 \geq |u_0|$. Set two cases of initial states of followers as: case 1: $x(0) = [x_1(0), x_2(0), x_3(0)]^T = [3, -1, 5]^T$, $v(0) = [v_1(0), v_2(0), v_3(0)]^T = 0 \times 1_3$ and case 2: $x(0) = [x_1(0), x_2(0), x_3(0)]^T = [30, -10, 50]^T$, $v(0) = [v_1(0), v_2(0), v_3(0)]^T = 0 \times 1_3$. The input disturbances of the followers are selected as $d_1 = 1.3\sin(2t)$, $d_2 = 0.9\cos(t)$, $d_3 = 1.8\sin(2t)$. To illustrate the result of Theorem 1 with protocol (4), we choose the parameters $\alpha = 10$, $\beta = 3 > d_{\max} + l = 2.05$, $k_1 = 2$, $k_2 = 1.2$, $\phi = 0.6$, $\varphi = 1.1$. Furthermore, $\lambda_{\min} = 0.2679$, $\lambda_{\max} = 3.7321$ can be obtained from the communication topology in Figure 1. According to (9) and (12), one can get that $T_1 = 0.6707s$, $T_2 = 38.0226s$, respectively. Thus, the total upper bound of settling time $T_{\max} = 38.6933s$ is the sum of T_1 and T_2 based on Theorem 1. For two different initial states, the sliding surfaces are depicted in Figure 2 and Figure 3, respectively. Figure 2 and Figure 3 show that the tracking error outside the sliding mode surface will quickly approach the sliding mode surface. Once it reaches the sliding mode surface, the tracking error will remain on the sliding mode surface, i.e., $S = 0$ can be always held.

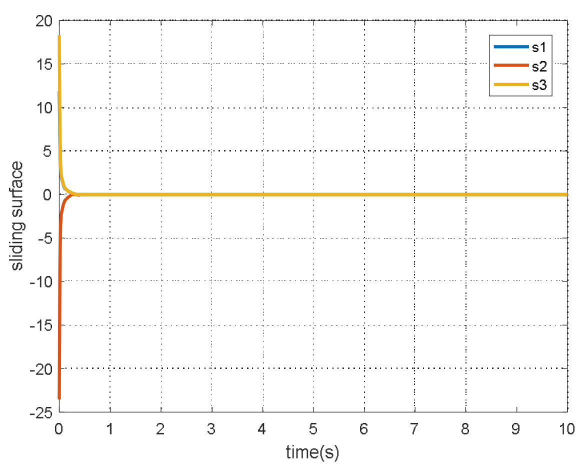

Figure 2 Time evolution of the sliding surface S for case 1

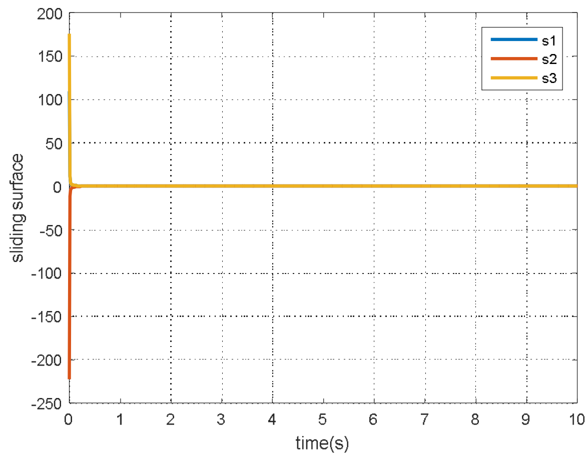

Figure 3 Time evolution of the sliding surface S for case 2

Next, for case 1, the consensus tracking results for position and velocity states of second-order multi-agent systems are drawn in Figure 4 and Figure 5, respectively. Figure 4 and Figure 5 show that consensus tracking is achieved at about $3.5s < T_{\max} = 38.6933s$, which shows the conservatism of the estimate for the settling time.

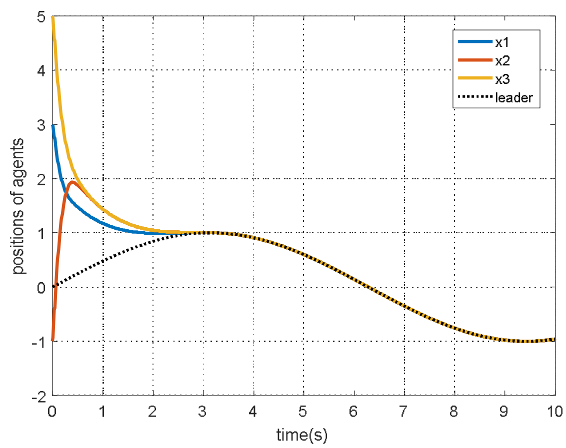

Figure 4 Consensus tracking results for position x for case 1

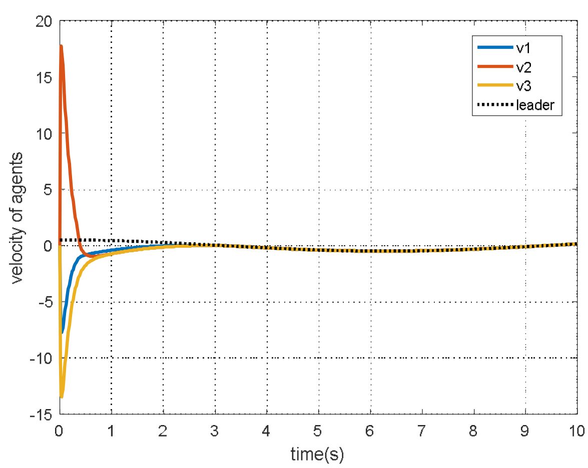

Figure 5 Consensus tracking results for velocity v for case 1

For case 2, the consensus tracking results are drawn in Figure 6 and Figure 7, respectively. Figure 6 and Figure 7 show that consensus tracking is achieved at about $6.5s < T_{\max} = 38.6933s$.

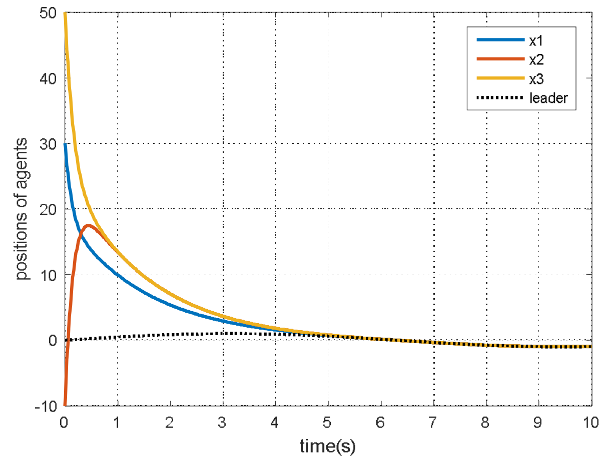

Figure 6 Consensus tracking results for position x for case 2

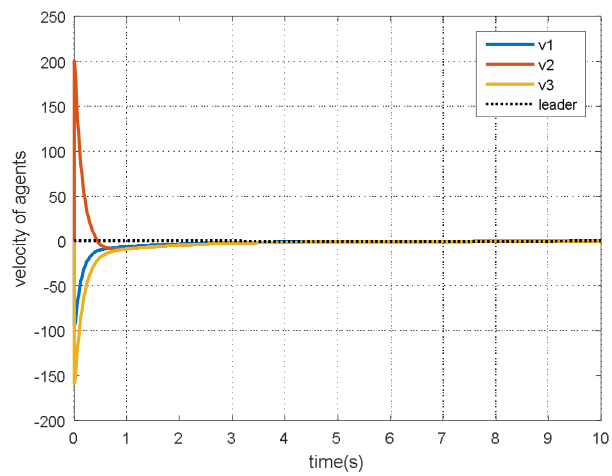

Figure 7 Consensus tracking results for velocity v for case 2

The simulation results from these two cases show that the state of second-order multi-agent systems with bounded disturbances can track the state of the leader in a fixed time under the control protocol (4) and the settling time is uniformly bounded by $T_{\max} = 38.6933s$ for different initial states of agents. Thus, the performance described in this paper is confirmed.

5. CONCLUSIONS

In the paper, fixed-time consensus tracking problem for second-order multi-agent systems with bounded disturbances has been discussed. A novel distributed nonlinear protocol has been proposed under which each follower can track the trajectory of the leader through local information exchange in a fixed time. Meanwhile, a new sliding mode manifold has been proposed to analyze the stability of the closed-loop error system. Based on a sliding mode control mechanism, the tracking errors can reach the sliding mode surface in a fixed time. Once the sliding mode surface is reached, it is proved that the tracking errors can arrive at the origin in a fixed time along the sliding surface based on theoretical analysis. Particularly, based on fixed-time stability theory, the convergence time for second-order multi-agent systems is bounded by a positive constant which depends only on controller parameters and network parameters regardless of initial conditions. Thus, this makes it possible to predesign the upper bound of convergence time by properly choosing the control parameters. The numerical simulations have been provided to demonstrate that the fixed-time consensus tracking is achieved under the proposed control protocol for second-order multi-agent systems with bounded disturbances. For future works, we will extend the results of this paper to directed fixed topologies, switching topologies and heterogeneous multi-agent systems.

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