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IDEALS AND IMPLICATIVE *WI*-IDEALS OF LATTICE WAJSBERG ALGEBRAS

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ABSTRACT

In the present paper, we consider ideal of lattice Wajsberg algebra and investigate some related properties. Also, we introduce the notion of implicative WI-ideal of lattice Wajsberg algebra. Further, we discuss some of its characterizations, and obtain some properties of lattice H-Wajsberg algebra. Moreover, we analyze the relationship between WI-ideal and implicative WI-ideal of lattice Wajsberg algebra as well as in lattice H-Wajsberg algebra. Finally, we investigate the relationship between implicative WI-ideal and implicative filter of lattice Wajsberg algebra.

Keywords: Wajsberg algebra; Lattice Wajsberg algebra; Lattice *H*-Wajsberg algebra; *WI*-ideal; Lattice ideal; Ideal; Implicative *WI*-ideal.

Mathematical Subject classification 2010: 03G10, 03G20, 03G25, 06B10.

1. INTRODUCTION

Lukasiewicz logic is a non-classical many valued logic. In 1935 Wajsberg [9] announced that he had a proof of Lukasiewicz conjecture. But, Wajsberg's proof was not published. Mathematical proof of Lukasiewicz conjecture was given by Rose and Rosser [8] in 1958. In 1981, Komori [7] introduced his CN-algebra as an algebraic counterpart of many valued Lukasiewicz propositional logic. The theory of Komori's CN-algebra was developed by Font et al. [1] in 1984 and called them as Wajsberg algebra and also introduced lattice structure of Wajsberg algebra. In 1990, Hernando Gaitan [2] introduced ideal of Wajsberg algebras. The authors [3] introduced the notion of Wajsberg implicative ideal (*WI*-ideal) of lattice Wajsberg algebras and discussed some related properties. Moreover, the authors [4], [5] and [6] introduced the notions of fuzzy *WI*-ideal, normal fuzzy *WI*-ideal, intuitionistic *WI*-ideal and annihilator of lattice Wajsberg algebras, and also investigated their properties with suitable illustrations. In this paper, we consider ideal of lattice Wajsberg algebra and investigate some related properties. Also, we introduce the notion of implicative *WI*-ideal of lattice Wajsberg algebra. Further, we discuss some of its characterizations and we obtain some properties of lattice *H*-Wajsberg algebra. Moreover, we analyze the relationship between *WI*-ideal and implicative *WI*-ideal of lattice Wajsberg algebra as well as in lattice *H*-Wajsberg algebra. Finally, we obtain the relationship between implicative *WI*-ideal and implicative filter of lattice Wajsberg algebra.

2. PRELIIMINARIES

In this section, we recollect some basic definitions and their properties which are necessary to develop our main results.

Definition 2.1[1]. Let $(A, \rightarrow, *, 1)$ be an algebra with quasi complement "*" and a binary operation "$\rightarrow$" is called a Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in A$,

(i) $1 \rightarrow x = x$

(ii) $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$

(iii) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$

(iv) $(x^* \rightarrow y^*) \rightarrow (y \rightarrow x) = 1$

Proposition 2.2[1]. A Wajsberg algebra (A, $\rightarrow$, $*$, 1) satisfies the following axioms for all $x, y, z \in A$,

(i) $x \rightarrow x = 1$

(ii) If $(x \rightarrow y) = (y \rightarrow x) = 1$ then $x = y$

(iii) $x \rightarrow 1 = 1$

(iv) $(x \rightarrow (y \rightarrow x)) = 1$

(v) If $(x \rightarrow y) = (y \rightarrow z) = 1$ then $x \rightarrow z = 1$

(vi) $(x \rightarrow y) \rightarrow ((z \rightarrow x) \rightarrow (z \rightarrow y)) = 1$

(vii) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$

(viii) $x \rightarrow 0 = x \rightarrow 1^* = x^*$

(ix) $(x^*)^* = x$

(x) $(x^* \rightarrow y^*) = y \rightarrow x$.

Definition 2.3[1]. A Wajsberg algebra A is called a lattice Wajsberg algebra if it satisfies the following conditions for all $x, y \in A$,

(i) The partial ordering "$\leq$" on a lattice Wajsberg algebra A, such that $x \leq y$ if and only if $x \rightarrow y = 1$

(ii) $(x \vee y) = (x \rightarrow y) \rightarrow y$

(iii) $(x \wedge y) = ((x^* \rightarrow y^*) \rightarrow y^*)^*$.

Note. From the definition 2.3 an algebra (A, $\vee$, $\wedge$, $*$, 0, 1) is a lattice Wajsberg algebra with lower bound 0 and upper
bound 1.

Proposition 2.4[1]. A lattice Wajsberg algebra (A, $\rightarrow$, $*$, 1) satisfies the following axioms for all $x, y, z \in A$

(i) If $x \leq y$ then $x \rightarrow z \geq y \rightarrow z$ and $z \rightarrow x \leq z \rightarrow y$

(ii) $x \leq y \rightarrow z$ if and only if $y \leq x \rightarrow z$

(iii) $(x \vee y)^* = (x^* \wedge y^*)$

(iv) $(x \wedge y)^* = (x^* \vee y^*)$

(v) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$

(vi) $x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z)$

(vii) $(x \rightarrow y) \vee (y \rightarrow x) = 1$

(viii) $x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)$

(ix) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$

(x) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(xi) $(x \wedge y) \rightarrow z = (x \rightarrow y) \rightarrow (x \rightarrow z)$.

Definition 2.5[3]. The lattice Wajsberg algebra A is called a lattice H-Wajsberg algebra if $x \vee y \vee ((x \wedge y) \rightarrow z) = 1$ for all $x, y, z \in A$. In a lattice H-Wajsberg algebra A, the following hold,

(i) $x \rightarrow (x \rightarrow y) = (x \rightarrow y)$

(ii) $x \rightarrow (y \rightarrow z) = (x \rightarrow y) \rightarrow (x \rightarrow z)$.

Definition 2.6[1]. Let L be a lattice. An ideal I of L is a nonempty subset of L is called a lattice ideal if it satisfies the following axioms for all $x, y \in I$,

(i) $x \in I, y \in L$ and $y \leq x$ imply $y \in I$

(ii) $x, y \in I$ implies $x \vee y \in I$.

Definition 2.7[3]. Let *A* be a lattice Wajsberg algebra. Let *I* be a nonempty subset of *A*. Then *I* is called *WI*-ideal of lattice Wajsberg algebra *A* satisfies for all $x, y \in A$,

(i) $0 \in I$

(ii) $(x \to y)^* \in I$ and $y \in I$ imply $x \in I$.

Definition 2.8[3]. Let A_1 and A_2 be lattice Wajsberg algebras, $f: A_1 \to A_2$ be a mapping from A_1 to A_2, if for any $x, y \in A_1$ $f(x \to y) = f(x) \to f(y)$ holds, then *f* is called an implication homomorphism from A_1 to A_2. If *f* is an implication homomorphism and satisfies,

(i) $f(x \vee y) = f(x) \vee f(y)$

(ii) $f(x \wedge y) = f(x) \wedge f(y)$

(iii) $f(x^*) = (f(x))^*$ for all $x, y \in A_1$.

Then, *f* is called a lattice implication homomorphism from A_1 to A_2.

Definition 2.9[2]. A non-empty subset *I* of a Wajsberg algebra *A*, is an ideal if it satisfies the following for all $x, y \in A$,

(i) $0 \in I$

(ii) $x \in I$ and $y \le x$ imply $y \in I$

(iii) $f(x^*) = (f(x))^*$ for all $x, y \in A_1$.

Definition 2.10[1]. Let *A* be a Wajsberg algebra, a subset *F* of *A* is called an implicative filter of *A*, if it satisfies the following axioms for all $x, y \in A$,

(i) $1 \in F$

(ii) $x \in F$ and $x \to y \in F$ imply $y \in F$.

3. MAIN RESULT

3.1 Ideal of lattice Wajsberg algebras

In this section, we consider ideal of lattice Wajsberg algebra and investigate some properties of ideal.

Proposition 3.1.1. Intersection of two ideals of lattice Wajsberg algebra *A* is an ideal.

Proof. Let K_1 and K_2 be two ideals of lattice Wajsberg algebra *A*.

Since $0 \in K_1$ and $0 \in K_2$ imply $0 \in K_1 \cap K_2$.

Therefore $K_1 \cap K_2$ is non-empty, if $x \in K_1 \cap K_2$ and $y \le x$

Then we have, $x \in K_1$ and $y \le x$ (1)

$x \in K_2$ and $y \le x$ (2)

Since K_1 and K_2 are ideals of *A*. From (1) and (2) $y \in K_1$ and $y \in K_2$. Then, $y \in K_1 \cap K_2$.

Since $x, y \in K_1 \cap K_2$ imply $x^* \to y \in K_1 \cap K_2$.

Hence, we get intersection of two ideals of lattice Wajsberg algebra *A* is an ideal. ■

Remark 3.1.2. Union of two ideals of lattice Wajsberg algebra *A* need not be an ideal of *A*.

Proposition 3.1.3. Let *A* be a lattice Wajsberg algebra every ideal of *A* is a lattice ideal.

Proof. Let *F* be an ideal of lattice Wajsberg algebra *A*. From (ii) of definition 2.9 shows that *F* satisfies (i) of definition 2.6.

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Now, $(x\vee y)^* \to y = ((x\to y)\to y)^* \to y)$

$= ((y^* \to (x\to y)^*) \to y)$

$= ((y^* \to (y^* \to x^*)) \to y)$

$= ((x^* \to (x^* \to y^*)) \to y)$

$= ((x\wedge y)^* \to y)$

$= ((x^* \vee y^*) \to y)$

$= (x^* \to y) \wedge (y^* \to y)$

$= (x^* \to y) \in F$

Thus, we get $(x\vee y)^* \to y = (x^* \to y) \in F$. Since F is an ideal, $(x\vee y)^* \to y \in F$ imply $(x\vee y) \in F$ and $y \in F$. From (iii) of definition 2.9, we have F is a lattice ideal. ■

Example 3.1.4. Let $A = \{0, u, v, w, x, y, z, 1\}$ be a set with Figure (1) as a partial ordering. Define a quasi complement "$*$" and a binary operation "$\to$" on A as in tables (1) and (2).

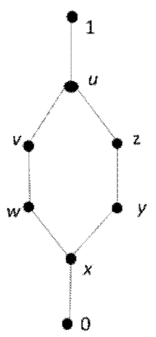

Figure (1)

x	x^*
0	1
U	X
V	Y
W	Z
X	U
Y	V
Z	W
1	0

Table (1)

$\to$	0	U	V	W	X	Y	Z	1
0	1	1	1	1	1	1	1	1
U	X	1	U	U	Z	Z	Z	1
V	Y	1	1	1	U	Z	Z	1
W	Z	1	1	1	1	Z	Z	1
X	U	1	1	1	1	1	1	1
Y	V	1	U	U	U	1	1	1
Z	W	1	U	U	U	U	1	1
1	0	U	V	W	X	Y	Z	1

Table (2)

Define $\vee$ and $\wedge$ operations on A as follow:

$(x\vee y) = ((x\to y)\to y)$,

$(x\wedge y) = ((x^* \to y^*) \to y^*)^*$ for all $x, y \in A$.

Then, A is a lattice Wajsberg algebra. It is easy to verify that, $I_1 = \{0, u\}$ is an ideal of A and also a lattice ideal of A.

Proposition 3.1.5. In lattice H-Wajsberg algebra A every lattice ideal is an ideal.

Proof. Let A be lattice H-Wajsberg algebra. Let I be a lattice ideal of A for all $x, y, z \in A$.
Then $x \in I$, $y \in A$ and $y \le x$ imply $y \in I$, for $x, y \in I$ imply $x \vee y \in I$.

Since I is a lattice ideal, it satisfies (ii) of definition 2.9. Also, $x, y \in I$ which imply $x^* \to y \in I$.

Hence, we get I is an ideal. ■

Proposition 3.1.6. In lattice *H*-Wajsberg algebra *A* every ideal is a *WI*-ideal.

Proof. Let F be an ideal of lattice Wajsberg algebra *A*. Then, we have $0 \in F$, $x \in F$ and $y \le x$ imply $y \in F$ and $x, y \in F$ imply $x^* \to y \in F$.

Now $((x \to y)^* \to y) = (y^* \to (x \to y))$ [from (x) of proposition 2.2]

$$= ((y^* \to x) \to (y^* \to y))$$

$$= ((x^* \to y) \to (y^* \to y))$$

Since F is an ideal $x^* \to y \in F$. Hence $((x^* \to y) \to (y^* \to y)) \in F$ also $((x \to y)^* \to y) \in F$.

Therefore, $(x \to y)^* \in F$ and $y \in F$ imply $x \in F$ .Hence, we get F is a *WI*-ideal. ■

In example 3.1.4, $I_1 = \{0, u\}$ is also a *WI*-ideal.

3.2 Implicative *WI*-ideal of lattice Wajsberg algebras

In this section, we introduce the concept of implicative *WI*-ideal of lattice Wajsberg algebra and we obtain some of its properties with illustrations.

Definition 3.2.1. Let I be a nonempty subset of lattice Wajsberg algebra *A*. Then, I is said to be an implicative *WI*-ideal of *A*, if it satisfies the following conditions for all $x, y, z \in A$,
(i) $0 \in I$
(ii) $(y \to z)^* \in I$ and $((x \to y)^* \to z^*) \in I$ imply $(x \to z)^* \in I$.

Proposition 3.2.2. If I is an implicative *WI*-ideal of lattice Wajsberg algebra *A* then I is a *WI*-ideal of *A*.

Proof. Let I be an implicative *WI*-ideal of lattice Wajsberg algebra *A*,then $0 \in I$, $(y \to z)^* \in I$ and $((x \to y)^* \to z^*) \in I$ imply $(x \to z)^* \in I$. If $y \in I$ and $(x \to y)^* \in I$ for all $x, y \in A$, then $(y \to 0)^* = (y^*)^* = y \in I$ and $((x \to y)^* \to 0)^* = (((x \to y)^*)^*)^* = (x \to y)^* \in I$. Since I is an implicative *WI*-ideal of lattice Wajsberg algebra *A*. Which follows that $x = (x^*)^* = (x \to 0)^* \in I$ .Hence, we have I is a *WI*-ideal of *A*. ■

Example 3.2.3. Let $A = \{0, p, q, r, s, t, 1\}$ be a set with Figure (2) as a partial ordering. Define a quasi complement "$*$" and a binary operation "$\to$" on *A* as in tables (3) and (4).

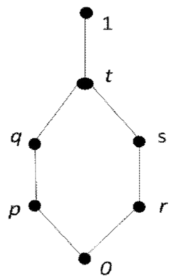

Figure (2)

x	x^*
0	1
P	S
Q	S
R	Q
S	Q
T	0
1	0

Table (3)

$\to$	0	P	Q	R	S	T	1
0	1	1	1	1	1	1	1
P	S	1	1	S	S	1	1
Q	S	T	1	S	S	1	1
R	Q	Q	Q	1	1	1	1
S	Q	Q	Q	T	1	1	1
T	0	Q	Q	S	S	1	1
1	0	P	Q	R	S	T	1

Table (4)

Define $\vee$ and $\wedge$ operations on A as follow:
$(x \vee y) = (x \to y) \to y$,
$(x \wedge y) = ((x^* \to y^*) \to y^*)^*$ for all $x, y \in A$.
Then, A is a lattice Wajsberg algebra. It is easy to verify that, $I_2 = \{0, q, s, 1\}$ is an implicative *WI*-ideal of lattice Wajsberg algebra *A*and also a *WI*-ideal of *A*.

Proposition3.2.4. If A is a lattice *H*-Wajsberg algebra then the following equality holds $(x \to y)^* \to z = (x \to z)^* \to (y \to z)^*$ for all $x, y, z \in A$.

Proof.Let *A* be a lattice *H*-Wajsberg algebra

$$(x \to y)^* \to z = z^* \to (x \to y)$$
$$= z^* \to (y^* \to x^*) \quad \text{[from (x) of proposition 2.2]}$$
$$= (z^* \to y^*) \to (z^* \to x^*) \quad \text{[from (ii) of definition 2.5]}$$
$$= (y \to z) \to (x \to z) \quad \text{[from (x) of proposition 2.2]}$$

Hence, $(x \to y)^* \to z = (x \to z)^* \to (y \to z)^*$. ■

Proposition 3.2.5. Let *A* be a lattice Wajsberg algebra, subset $I = \{0\}$ of *A* is a *WI*-ideal of *A*.

Proof. If $(x \to y)^* \in I$, $y \in I$ for all $x, y \in A$. Then, $y = 0$, $(x \to y)^* = 0$ imply $(x \to y) = 1$, hence $x \le y = 0$. Then, $x = 0$ which means that $x \in I$. Hence, $I = \{0\}$ of *A* is a *WI*-ideal of *A*. ■

Proposition 3.2.6. Every *WI*-ideal of a lattice *H*-Wajsberg algebra is an implicative *WI*-ideal of lattice Wajsberg algebra *A*.

Proof. Let *A* be a lattice *H*-Wajsberg algebra. Let *I* be a *WI*-ideal of *A* for all $x, y, z \in A$.

If $(y \to z)^*, ((x \to y)^* \to z)^* \in I$ then from proposition 3.2.4 that $((x \to z)^* \to (y \to z)^*)^* = ((x \to y)^* \to z)^* \in I$, since I is a *WI*-ideal of A $(x \to z)^* \in I$ .Hence, I is an implicative *WI*-ideal of lattice Wajsberg algebra *A*. ■

Proposition 3.2.7. Let A is a lattice Wajsberg algebra then the following equality holds
$((x \to (y \to z)) \to (x \to z))^* \to ((x \to y) \to (x \to z)) = 1$ for all $x, y, z \in A$.

Proof. $((x \to (y \to z)) \to (x \to z))^* \to ((x \to y) \to (x \to z))$

$$= ((y \to (x \to z)) \to (x \to z))^* \to ((x \to y) \to (x \to z))$$
$$= (y \vee (x \to z))^* \to ((x \to y) \to (x \to z))$$
$$= (y^* \wedge (x \to z)^*) \to ((x \to y) \to (x \to z))$$
$$\geq y^* \to ((x \to y) \to (x \to z))$$
$$= y^* \to ((y^* \to x^*) \to (z^* \to x^*))$$
$$= y^* \to (z^* \to (y^* \to x^*) \to x^*)$$
$$= y^* \to (z^* \to (y^* \vee x^*))$$
$$= z^* \to (y^* \to (y^* \vee x^*))$$
$$= z^* \to 1 = 1$$

Hence, $((x \to (y \to z)) \to (x \to z))^* \to ((x \to y) \to (x \to z)) = 1$. ■

Proposition 3.2.8. Let A is a lattice Wajsberg algebra then for all $x, y, z \in A$
$(x \to (y \to z)) \to ((x \to y) \to (x \to z)) = x \vee y \vee ((x \wedge y) \to z)$

Proof. For all $x, y, z \in A$

$$(x \to (y \to z)) \to ((x \to y) \to (x \to z)) = (x \to (y \to z)) \to ((y \to z) \vee (x \to z))$$
$$= ((x \to (y \to z)) \to (y \to z)) \vee ((x \to (y \to z)) \to ((x \to z)))$$
$$= x \vee ((y \to z)) \vee y \vee ((x \to z))$$
$$= x \vee y \vee ((x \wedge y) \to z)$$

Hence, we get $(x \to (y \to z)) \to ((x \to y) \to (x \to z)) = x \vee y \vee ((x \wedge y) \to z)$. ■

Proposition 3.2.9. Let A be a lattice *H*-Wajsberg algebra if and only if one of the following conditions holds for all $x, y, z \in A$

(i) $x \to (y \to z) = (x \to y) \to (x \to z)$

(ii) $(x \to (y \to z)) \to ((x \to y) \to (x \to z)) = 1$

(iii) $(x \to (y \to z)) = (x \wedge y) \to z$

(iv) $(x \to (x \to y) = x \to y$

(v) $(x \to y) \to x = x$

(vi) $(x \vee x^*) = 1$.

Proof. By the definition 2.5 and proposition 3.2.8

$$x \to (y \to z) \leq (x \to y) \to (x \to z) \tag{3}$$

on the other hand,

$$((x \to y) \to (x \to z)) \to (x \to (y \to z)) = ((x \to z) \vee (y \to z)) \to (x \to (y \to z))$$
$$= ((x \to z) \to (x \to (y \to z))) \wedge ((y \to z) \to (x \to (y \to z)))$$

$=(y \to ((x \to z) \to (x \to z))) \wedge (x \to ((y \to z) \to (y \to z)))$
$=1$

That is, $(x \to y) \to (x \to z) \leq x \to (y \to z)$ (4)

From (3) and (4), we get $x \to (y \to z) = (x \to y) \to (x \to z)$.
Hence, (i) is proved and the proof of (ii), (iii), (iv), (v), (vi) are straight forward. ■

Proposition 3.2.10. Let *A* be a lattice Wajsberg algebra and *A* be a lattice *H*-Wajsberg algebra if and only if $I = \{0\}$ is an implicative *WI*-ideal of *A*.

Proof. Let *A* be a lattice *H*-Wajsberg algebra then it follows from proposition 3.2.6 and proposition 3.2.5 that $I = \{0\}$ is an implicative *WI*-ideal of *A*.
If *A* is lattice Wajsberg algebra and $I = \{0\}$ is an implicative *WI*-ideal of lattice Wajsberg algebra *A*, this implies $x, y, z \in A$ $((x \to z) \to ((x \to y) \to (x \to z)))^* = 1^* = 0 \in I$, on the other hand, from proposition 3.2.6
$(((x \to (y \to z)) \to ((x \to z))^* \to (x \to y) \to (x \to z)))^* = 1^* = 0 \in I$,
I is an implicative *WI*-ideal of *A*, so $(((x \to (y \to z)) \to ((x \to y) \to (x \to z)))^* \in I$.
That is $(x \to (y \to z)) \to ((x \to y) \to (x \to z)) = 1$
From the above result and proposition 3.2.9.we say that, *A* is a lattice *H*-Wajsberg algebra. ■

Proposition 3.2.11. If *A* is a lattice *H*-Wajsberg algebra if and only if every *WI*-ideal of *A* is an implicative *WI*-ideal of *A*.

Proof. This result can be proved by proposition 3.2.6, proposition 3.2.10 and proposition 3.2.5. ■

Proposition 3.2.12. Let A_1, A_2 are two lattice Wajsberg algebras $f : A_1 \to A_2$ be lattice implication homomorphism. If I is an implicative *WI*-ideal of A_2 then $f^{-1}(I)$ is an implicative *WI*-ideal of A_1.

Proof. If $(y \to z)^* \in f^{-1}(I)$ and $((x \to y)^* \to z)^* \in f^{-1}(I)$ for all $x, y, z \in A$
then, $(f(y) \to f(z))^* = f((y \to z)^*) \in I$ and
$(f(x) \to f(y))^* \to f(z))^* = f(((x \to y)^* \to z)^*) \in f^{-1}(I)$
since I is an implicative *WI*-ideal of A_2.
$f((x \to z)^*) = (f(x) \to f(z))^* \in I$ that is $(x \to z)^* \in f^{-1}(I)$.
Thus, $f^{-1}(I)$ is an implicative *WI*-ideal of A_1. ■

Proposition 3.2.13. Let *A* be a lattice Wajsberg algebra and *I* be a subset of *A*. Define $I^* = \{x^* / x \in I\}$ is an implicative *WI*-ideal of *A* if and only if I^* is an implicative filter of *A*.

Proof. Let *I* be an implicative *WI*-ideal of *A*. Now $1 = 0^* \in I^*$ since $0 \in I$ for all $x, y, z \in A$.
If $x \to y$ and $x \to (y \to z) \in I^*$ then $y^* \to x^* = x \to y \in I^*$ and
$((z^* \to y^*) \to x^*) = (x \to (y \to z)) \in I^*$, which implies that $(y^* \to x^*)^* \in I$,
$((z^* \to y^*) \to x^*)^* \in I$. It follows that $(x \to z)^* = (z^* \to x^*)^* \in I$.
Since *I* is an implicative *WI*-ideal of *A*. Consequently, $(x \to z) \in I^*$.
Thus, I^* is an implicative filter of *A*.

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Conversely, if I^* is an implicative filter of *I*, $0 = 1^* \in I$. Since $1 \in I^* I$ for all $x, y, z \in I$.

If $(y \to z)^*$ and $((x \to y)^* \to z)^* \in I$ .Then, we have

$(z^* \to y^*)^* = (y \to z)^* \in I$ and $(z^* \to (y^* \to x^*))^* = ((x \to y)^* \to z)^* \in I$ namely $(z^* \to y^*) \in I^*$ and $z^* \to (y^* \to x^*) \in I^*$ then $(x \to z) = (z^* \to x^*) \in I^*$. Since, I^* is an implicative filter of *A* equivalently $(x \to z)^* \in I$. So, I is an implicative ideal of *A*. ■

4. Conclusion

In the present paper, we have considered ideal of lattice Wajsberg algebra and investigated some related properties. Also, we have introduced the notion of implicative *WI*-ideal of lattice Wajsberg algebra. Further, we have discussed some of its characterizations and we have established some properties of lattice *H*-Wajsberg algebra. Moreover, we have examined the relationship between *WI*-ideal and implicative *WI*-ideal of lattice Wajsberg algebra as well as in lattice *H*-Wajsberg algebra. Finally, we have proved the relationship between implicative *WI*-ideal and implicative filter of lattice Wajsberg algebra.

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