Distributed Event-Triggered Consensus Tracking of First-Order Multi-Agent Systems with a Virtual Leader

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ABSTRACT

This paper proposes a novel event-triggered consensus tracking protocol for first-order multi-agent systems with a virtual leader. The event detector is designed based on the local sampled-data, and information exchange happens only at corresponding event instants. The Lyapunov function method is used to derive the sufficient conditions for consensus tracking. The bound of the inter-event time is ensured, and thus Zeno-behavior is avoided. Finally, numerical simulations are provided to illustrate the effectiveness of the proposed protocol.

Keywords: consensus tracking, event-triggered, multi-agent systems, sampled-data

1. INTRODUCTION

Multi-agent systems are composed of multiple autonomous agents communicating with each other to complete their cooperative control objectives such as swarming, flocking, formation, rendezvous, tracking, and so on. In the past decade, analysis and synthesis of multi-agent systems have been extensively studied by various researchers from multiple areas such as mathematics, physics, management science, and computer science. As a fundamental issue in distributed cooperative control of multi-agent systems, consensus means that all agents using distributed control inputs called consensus protocols eventually reach a common value [1-5].

As the computer technology develops, an agent may be equipped with an embedded microprocessor to obtain information and update control input. However, since the power supply of an embedded microprocessor is constrained, it is urgent for researchers to design a proper consensus algorithm for multi-agent systems such that the energy consumption can be reduced while the control performance is guaranteed. A traditional method is time-triggered control [6-8], where the control execution and information transmission are in a periodic pattern. Although this method is effective for energy saving to some extent, the sampling period is often chosen according to the worst-case case, which may lead to unnecessary energy waste. Motivated by this conflict, the event-triggered control strategy has been subsequently proposed. In the event-triggered fashion, controller actuation only happens at some specific instants determined by the predefined event condition rather than continuously or in a periodic pattern. It was proved in Ref. [9] that compared with time-triggered control the event-triggered control is more preferred in practice since it eliminates unnecessary computation and communication.

Recently, event-triggered control has been extensively studied in interconnected systems [10-11]. In Ref. [12], the event-triggered mechanism is introduced into multi-agent systems, where both centralized and distributed event-triggered control strategies for each agent to determine control updates are proposed. Subsequently, the event-triggered control strategy is applied to the consensus problems of high-order multi-agent systems. In Ref. [13] a time-dependent event strategy is applied to the consensus problems for three kinds of multi-agent systems, where agents decide themselves when to communicate with their neighbors based only on local information, which quite reduces communication consumption. The homologous property is also preserved in Ref. [14], where continuous communication among agents is replaced with a discrete mode that information transmission only happens at corresponding event instants. Heterogeneous and general linear multi-agent systems via event-triggered control are investigated in Refs. [15-17]. Moreover, the event-triggered control strategies are proved to be applicable for multi-agent systems with more complicated cases, such as packet dropout [18], communication delays [13], and noises [19]. In essence, the focus of event condition contains two factors: lower frequency of control updates and less information transmission among agents. However, when it comes to a consensus tracking case, conditions are more complicated than those in the consensus case. Recently, there are some studies on event-triggered consensus tracking of multi-agent systems [20-23]. In Ref. [20], the consensus tracking problems for multi-agent systems with and without communication delays were considered, where each follower was assumed to be a single-integrator. In Ref. [21], a
distributed event-triggered control strategy for second-order multi-agent system was proposed, where the control input incorporated continuous measurement of leader. In Ref. [22], an event-triggered strategy and a distributed estimation were proposed for the dynamic tracking control of second-order multi-agent systems while the communication among agents needed to be carried out constantly. In Ref. [23], Li et al. considered the event-triggered consensus tracking of second-order multi-agent systems with fixed and switching topologies, where the high-frequency control updating happens if the states of agents are close. In the above references [20-23] about event-triggered consensus tracking of multi-agent systems, there is a common fact that the event condition is assumed to be continuously examined, which implies a great amount of energy waste. Motivated by this observation, this paper proposes a novel distributed event-triggered consensus tracking strategy, where continuous information is replaced with sampled data in event detection for lower communication. It is shown that the parameters of event condition can be selected such that the first-order multi-agent systems with a virtual leader achieve consensus tracking. The remainder of this paper is organized as follows: Section 2 briefly introduces some concepts in algebraic graph theory and states the problem. Section 3 presents convergence analysis of the dynamics of each agent is described by

\[ \dot{x}_i(t) = u_i(t), \quad i \in \{1, 2, \ldots, N\}, \]  

(1)

where \( x_i \in \mathbb{R} \) and \( u_i \in \mathbb{R} \) represent the state and control input of agent \( i \), respectively. The dynamic of the virtual leader can be described as

\[ \dot{x}_v(t) = 0 \]  

(2)

where \( x_v \in \mathbb{R} \) represents the state of the virtual leader.

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1 Algebraic graph theory and some notations

The interaction topology of multi-agent systems can be modeled by an undirected graph \( G = (V, E, A) \), which consists of a vertex set \( V = \{v_i | i = 1, 2, \ldots, N\} \) representing \( N \) agents, an edge set \( E \subseteq V \times V \) meaning the communication links between agents, and a weighted adjacency matrix \( A \). An edge between \( v_i \) and \( v_j \) is denoted by \( e_{ij} = (v_i, v_j) \). The adjacency element \( a_{ij} \) associated with the edge \( e_{ij} \) is positive, i.e., \( e_{ij} \in E \Leftrightarrow a_{ij} > 0 \). Moreover, we assume \( a_{ii} = 0 \) for all \( i \in I \). For an undirected graph \( G \), the adjacency matrix \( A \) is symmetric, i.e., \( a_{ij} = a_{ji} \). The set of neighbors of vertex \( v_i \) is denoted by \( N_i = \{v_j \in V : e_{ij} \in E\} \). Correspondingly, the degree matrix \( D \) of \( G \) is defined as \( D = \text{diag}(d_1, d_2, \ldots, d_N) \) with \( d_i = \sum_{j=1}^{N} a_{ij} \). The Laplacian matrix of \( G \) is denoted by \( L = D - A \). A path from \( v_i \) and to \( v_j \) means a sequence of distinct edges \( (v_i, v_{i1}), (v_{i1}, v_{i2}), \ldots, (v_{i\ell}, v_j) \) in a graph. If there is a path between \( v_i \) and \( v_j \), then \( v_i \) and \( v_j \) are called connected. Graph \( G \) is connected if and only if there is a path between any two vertices. An important fact of \( L \) is that all row sums are zero and thus \( L \) has a right eigenvector \( 1_N \) associated with the zero eigenvalue, i.e., \( L1_N = 0_N \), where \( 1_N \) and \( 0_N \) denote the \( N \)-dimensional column vectors with all ones and zeros, respectively. If graph \( G \) is connected, \( L \) has one and only one zero eigenvalue and the remaining eigenvalues of \( L \) are all positive.

The following notations will be used throughout this paper. \( \mathbb{R}^N \) means the \( N \)-dimensional Euclidean space. For a vector \( x \in \mathbb{R}^N \), its Euclidean norm is denoted as \( \|x\| \). For a matrix \( A \in \mathbb{R}^{N \times N} \), the transpose matrix and the largest eigenvalue of \( A \) are denoted as \( A^T \) and \( \lambda_{\text{max}} \), respectively. \( A > 0 \) or \( A < 0 \) represents that the matrix \( A \) is positive definite or negative definite, respectively.

2.2 Problem statement

Consider multi-agent systems composed of \( N \) agents and one virtual leader. The dynamics of each agent is described by

\[ \dot{x}_i(t) = u_i(t), \quad i \in \{1, 2, \ldots, N\}, \]  

(1)

where \( x_i \in \mathbb{R} \) and \( u_i \in \mathbb{R} \) represent the state and control input of agent \( i \), respectively. The dynamic of the virtual leader can be described as

\[ \dot{x}_v(t) = 0 \]  

(2)

where \( x_v \in \mathbb{R} \) represents the state of the virtual leader.
Denote the sampling period and the kth event time of agent i by h and t'_k, respectively. Our purpose is to design an event-triggered control strategy to solve the consensus tracking problem while reducing the energy consumption. At first, we propose the event-triggered consensus tracking protocol for agent i as follows

\[ u_i(t) = -k_i \sum_{j \in N_i} a_{ij}(x_j(t'_i) - x_i(t'_i)) - k_i b_{i0}(x_i(t'_i) - x_0(t'_i)), \quad t \in [sh, (s+1)h) \]  

(3)

where the position state \( x_i(t'_i) \) represents the last measurements transmitted by agent i, the position state \( x_j(t'_i) \) represents the last measurements received from the neighbor agent j, the position state \( x_0(t'_i) \) represents the last measurements received from the virtual leader, \( k_i > 0 \) and \( k_i > 0 \) are two control gains to be determined later, and \( t'_i = \max \{t | t \in [t'_i, k = 0,1, \ldots], t < t'_i + sh \} \). Let \( B = \{b_{i0}, b_{20}, b_{30} \ldots b_{n0} \} \), where \( b_{i0} = 1 \) if agent i has access to the virtual leader, and \( b_{i0} = 0 \) otherwise.

Considering that the errors play key roles in event detection, we first define the following state measurement error and tracking error for agent i as

\[ \phi_i(t'_i + sh) = x_i(t'_i) - x_i(t'_i + sh), \quad s = 1,2, \ldots \]  

(4)

\[ \phi^i(t'_i + sh) = \tilde{x}_i(t'_i) - \tilde{x}_i(t'_i + sh), \quad s = 1,2, \ldots \]  

(5)

where \( \tilde{x}_i(t) = x_i(t) - x_0(t) \). Combining the definition of the state measurement error in Equations (4) and (5), we can propose the following event condition

\[ \|k_i \lambda_i \phi_i(t'_i + sh) + k_i b_{i0} \phi^i(t'_i + sh)\| > \frac{m_i}{a} (2 - 2h \lambda_{i,1} - \frac{1}{a}) \|\hat{w}_i(t)\|_2. \]  

(6)

where \( m_i \) and \( a \) are two positive constants to be determined later. The matrix \( M \) is defined as \( M = k_i L + k_i B \) and \( \hat{w}_i(t) = k_i \sum_{j \in N_i} a_{ij}(x_j(t'_i) - x_i(t'_i)) + k_i b_{i0}(x_i(t'_i) - x_0(t'_i)) \)

(7)

**Remark 1** It can be seen that the event condition (6) is designed based solely on the local errors and discrete neighbors states. Each agent computes \( \phi_i(t'_i + sh) \) and \( \phi^i(t'_i + sh) \) to check the event condition at every sampling time instant.

Once the inequality (6) is satisfied, which means an event occurs, agent i denotes event time by \( t'_i = t \), transmits its current state, i.e., \( x_i(t) = x_i(t'_i) \) to all of its neighbors, resets errors to zero, and actuates controller updates. Similarly, when agent i receives \( x_i(t) \) from its neighbor agent j at corresponding times \( t'_i \), agent i updates \( \hat{w}_i(t) \) by replacing \( x_j(t'_i) \) with \( x_i(t) \). Therefore, \( \hat{w}_i(t) \) is only updated at all corresponding event times \( t'_i \) and \( t'_i \). Thereby, the next event instant can be accordingly defined as follows

\[ t'_{i+1} = t'_i + sh \inf \{ sh : \|k_i \lambda_i \phi_i(t'_i + sh) + k_i b_{i0} \phi^i(t'_i + sh)\| > \frac{m_i}{a} (2 - 2h \lambda_{i,1} - \frac{1}{a}) \|\hat{w}_i(t)\|_2 \} \]

(8)

where \( t'_0 = 0 \) is the initial instant. Since the event condition of agent i remains invariant during a sampling period, the inter event times can be lowered bounded by sampling period \( h \). Thus, Zeno-behavior is absolutely excluded.

**Definition 1** The first-order multi-agent systems (1) with a virtual leader (2) are said to achieve consensus tracking if

\[ \lim_{t \rightarrow \infty} \|x_i(t) - x_0(t)\|_2 = 0, \quad i \in 1,2, \ldots, N \]  

(9)

holds for any initial conditions.

In the following, we will theoretically show the effectiveness of the protocol (3) with the condition (6) to guarantee the systems (1) with a virtual leader (2) to achieve consensus tracking. Before moving on, we need to provide the following lemma and assumption.

**Assumption 1** The network topology composed of \( N \) agents and a virtual leader is fixed and undirected, and there is a path from the virtual leader to each agent.

**Lemma 1** Let \( H = L + B \), where \( B = \text{diag} \{b_{i0}, b_{20}, \ldots, b_{n0} \} \) and \( b_{i0} \) is defined as above. If Assumption 1 is satisfied, the matrix \( H \) must be a positive definite matrix.\(^{[24]}\)
3. CONVERGENCE ANALYSIS

In this section, we will theoretically show the effectiveness of the protocol (3) with event condition (6) to assure the multi-agent systems (1) with a virtual leader (2) to achieve consensus tracking. At first, we need to transform the control input of agent $i$ into the following form

$$
\dot{x}_i(t) = -k_i \sum_{j \in N_i} a_j(x_i(t_j^+) - x_j(t_j^+)) - k_i b_{ij}(x_i(t_j^+) - x_j(t_j^+))
$$

$$
= -k_i \sum_{j \in N_i} a_j(x_i(t_j^+) - x_j(sh) - x_j(t_j^+) + x_i(sh) - x_j(sh))
$$

$$
- k_i b_{ij}(x_i(t_j^+) - x_j(sh) + x_i(sh) - x_j(t_j^+)), \ t \in [t_i^+ + sh, t_i^+ + (s + 1)h)
$$

Utilizing the definitions of state error and tracking error given in Equations (4) and (5), we can further revise the dynamics of agent $i$ as

$$
\dot{x}_i(t) = -k_i \sum_{j \in N_i} a_j(e_i(sh) - e_j(sh)) - k_i \sum_{j \in N_i} a_j(x_i(sh) - x_j(sh))
$$

$$
- k_i b_{ij} \phi_i(sh) - k_i b_{ij} \psi_i(sh), \ t \in [t_i^+ + sh, t_i^+ + (s + 1)h)
$$

Subsequently, the Equation (11) can be obtained in a compact form as

$$
\dot{x}(t) = -w(t) = -k_i L\phi_i(sh) - k_i L\psi_i(sh) - k_i B\psi_i(sh) - k_i B\phi_i(sh)
$$

where $\dot{x}(t) = (\dot{x}_1(t), \ldots, \dot{x}_N(t))^T$, $\phi_i(t) = (\phi_i(t), \ldots, \phi_i(t^+))^T$, $\psi_i(t) = (\psi_i(t), \ldots, \psi_i(t^+))^T$, and $w(t) = (w_1(t), \ldots, w_N(t))^T$. To reduce the clutter in the notation, we denote $e_i(sh) = -k_i L\phi_i(sh) - k_i B\phi_i(sh)$. Since the equation $L\dot{x}(sh) = Lx(sh)$ holds constantly based on the Assumption 1, we can obtain

$$
\dot{x}(t) = -w(t) = -e_i(sh) - M\dot{x}(sh)
$$

Now we provide the main results.

**Theorem 1** Under Assumption 1, the first-order multi-agent systems (1) with a virtual leader (2) applying the consensus tracking protocol (3) with the event condition (6) can achieve consensus tracking if and only if

$$
\frac{1}{h} (1 - \frac{1}{2a}) \lambda_M > 0, m_i < 1
$$

**Proof** Take a candidate Lyapunov function $V(t) = \frac{1}{2} x^T(t)Mx(t)$. According to the Lemma 1, it is apparent that $M$ is symmetric and positive such that Lyapunov function $V(t) > 0$. Combining the Taylor expansion of $\dot{x}(t)$ at $t = sh$ and Equation (12), we can obtain for any $t \in [sh, (s + 1)h)$

$$
\dot{V}(t) = \dot{x}^T(t)M\dot{x}(t)
$$

$$
= -x^T(t)Mw(t)
$$

$$
= -(\dot{x}^T(sh) - (t - sh)w^T(t))Mw(t)
$$

$$
\leq -x^T(sh)Mw(t) + hw^T(t)Mw(t)
$$

Considering $e^T(sh)M^T \dot{z}(t) \leq \frac{1}{2} e^T(sh)M^T e(sh) + \frac{1}{2} \dot{z}^T(t)M^T \dot{z}(t)$ and Equation (13), we can further simplify the above Equation (15) as

$$
\dot{V}(t) \leq -(w^T(t) - e^T(sh))w(t) + hw^T(t)Mw(t)
$$

$$
\leq -(1 + h\lambda_M)w^T(t)w(t) + e^T(t)w(t)
$$

$$
\leq -(1 + h\lambda_M + \frac{1}{2a})w^T(t)w(t) + \frac{a}{2} e^T(t)e(t)
$$

According to Remark 1, it is obvious that the error $\psi_i(sh)$ and $\phi_i(sh)$ will be reset to zero when the event condition (6) is satisfied. Consequently, we have following inequality

$$
\left\| k_i \lambda_i \phi_i(t_i^+ + sh) + k_i b_i \phi_i(t_i^+ + sh) \right\| \leq \frac{m_i}{a} \phi_i(2 - 2h\lambda_M - \frac{1}{a}) \left\| \dot{z}(t) \right\|
$$

Furthermore, we have
Thus, the function $\dot{V}(t)$ can be bounded as follows:

$$\dot{V}(t) = (1 - m_{\max})(-1 + h\lambda + \frac{1}{2a})w^T(t)w(t)$$

where $m_{\max} = \max\{m_i \mid i = 1, \ldots, N\}$. From Equation (19), we get that $\dot{V}(t)$ is negative semi-definite for $s \in \{0, 1, 2, \ldots\}$ and $t \in [sh, (s+1)h)$ as $w(t)^T w(t) \geq 0$. Since $V(t) \geq 0$, $\dot{V}(t) \leq 0$ implies that $V(t)$ has a finite limit and $\lim_{t \to \infty} V(t) = 0$.

At the same time, considering $\dot{V}(t) \leq (1 - m_{\max})(-1 + h\lambda + \frac{1}{2a})w^T(t)w(t)$, we have $\lim_{t \to \infty} w(t) = 0$. It follows from Equation (18) that $\lim_{t \to \infty} e(sh) = 0$. Combining the definition given by Equation (13), we can finally obtain

$$\lim_{t \to \infty} \dot{x}(sh) = 0$$

Thus, the proof of Theorem 1 is completed.

**Remark 2** It should be pointed out that in order to compute the event condition (6), each agent needs to be aware of partial global information, i.e., the largest eigenvalue of $L$. However, we can find an upper bound of $\lambda_i$ given by $\lambda_i \leq 2d_{\max} \leq 2(N-1)$ in Ref. [25]. Therefore, the constraint and event condition given by Equations (14) and (6) should be renewed as

$$\frac{1}{2k_i}(1 - \frac{1}{2a}) - k_i 1 > N$$

and

$$\|k_i(N-1)\phi(\tau_i + sh) + k_zb_{\phi}^{\phi}(\phi(\tau_i + sh))\|_2^2 \geq \rho \|\tilde{V}(t)\|^2$$

where $\rho = \frac{m_i}{a}(2 - 2h(2k_i(N-1) + k_z) - \frac{1}{a})$. Obviously, $\lim_{t \to \infty} \dot{V}(t) = 0$ can also be derived based on Equation (20) and (21). So the multi-agent systems (1) with a virtual leader (2) utilizing the event condition (21) can also reach consensus tracking.

4. NUMERICAL SIMULATIONS

In this section, numerical simulations are presented to illustrate the effectiveness of the proposed event-based control strategy. Here we consider multi-agent systems with three agents and one virtual leader, whose dynamics are described by Equations (1) and (2). The corresponding communication topology is shown in Figure 1.

**Figure 1** Network topology composed of three agents and one virtual leader

Without loss of generality, the weights of all the edges are assumed to be 1. Thus, we can obtain $L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ and $B = \text{diag}\{1 \ 0 \ 0\}$. By setting $k_1 = k_2 = 1$, the matrix $M$ is given by $M = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$. It is easily to obtain that $\lambda_2 = 3$ and $\lambda_M = 3.7321$. According to the conditions given by Equation (14), we choose the sampling period $h = 0.002$. We choose $a = 0.6$ according to $a > 0.5$. Correspondingly, $\frac{1}{a}(2 - 2h\lambda_M - \frac{1}{a}) \approx 1.92$. Assume the parameters of each event condition are $m_1 = 0.5$, $m_2 = 0.3$ and $m_3 = 0.8$. Without loss of generality, we choose the
initial states of the virtual leader and three agents as $x_i(t) = 0.5$ and $x(0) = [-2, 1, -0.39]^T$, respectively. Figure 2 shows that the states of all agents can converge to that of the virtual leader, in the sense that the first-order multi-agent systems (1) with the virtual leader (2) applying the protocol (3) with the event condition (6) can ultimately achieve consensus tracking. Here, we also provide the corresponding triggered instants of each agent in Figure 3. It can be seen that the Zeno-behavior can be absolutely avoided.

**Figure 2** The state evolution of the systems (1) with a virtual leader (2) applying the protocol (3)

**Figure 4** Event times of the systems (1) with a virtual leader (2) applying the event condition (6)

5. CONCLUSIONS

A novel event-triggered consensus tracking protocol for first-order multi-agent systems with a virtual leader is proposed in this paper. The control updates and communication under the proposed protocol only happen at even times, resulting in significantly reducing the energy consumption. The Lyapunov function method is used to derive the sufficient conditions guaranteeing consensus tracking. It is worth noting that all agents are equipped with an identical sampling period, so one of future works is devoted to the case of heterogeneous sampling periods.

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